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Skew Chromatic Index of Theta Graphs

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Abstract—A two edge coloring of a graph *G* is said be a skew edge coloring if no two edges of G are assigned the same unordered pair of colors. The least number of colors required for a skew edge coloring of G is called its skew chromatic index denoted by *s*(*G*). This article provides a method for skew edge coloring of uniform theta and quasi-uniform theta graphs as two component colorings by defining two mappings *f* and *g* from the edge set *E*(*G*) to the set of colors {1, 2, 3, …, *k*}. The minimum number of colors *k* which is known as the skew chromatic index is determined depending upon the number of edges of *G*. This work also proves that the bound on the skew chromatic index $s(G) \geq \max\{\Delta(G), k(|E(G)|)\}$ is sharp for the family of graphs considered for skew edge coloring.

Keywords—edge coloring; skew chromatic index; generalized theta graph; uniform theta graph; quasi-uniform theta graph

I. INTRODUCTION

Edge coloring problems in graphs arise in several computer science disciplines [1]. One of which is register allocation during code generation in a computer programming language compiler. This article considers a finite and simple graph *G* whose vertex set is *V* and edge set is *E*. A proper edge coloring of *G* is an assignment of colors to the edges such that adjacent edges are assigned different colors [2]. The least number of colors required for such an edge coloring of *G* is its chromatic index denoted by $\chi'(G)$. Vizing [3] has shown that for any simple graph *G*, $\chi'(G)$ is either $\Delta(G)$ or $\Delta(G)+1$, where $\Delta(G)$ is the highest degree of a graph *G*. Edge coloring problem has been proved to be *NP*-complete by Holyer [4]. This work includes skew edge coloring problems which are motivated from the analysis of skew Room squares [5]. Marsha introduced skew chromatic index and the related concepts [6]. The skew chromatic index *s*(*G*) have already been determined for comb, ladder, Mobius ladder and circular ladder graphs. [7, 8]. An assignment of color pairs of the form $\{a_i, b_i\}$ to each edge e_i of *G* such that (i) the a_i's and the b_i's separately form component edge colorings of *G* and (ii) all these pairs are distinct is called a skew edge coloring of *G*.
All the notations and definitions which appear in this article are the same as in [9].

II. LOWER BOUND ON SKEW CHROMATIC INDEX

The component coloring of a skew edge coloring is itself an edge coloring and therefore we have $s(G) \geq \chi'(G)$. But Vizing's theorem states that for any simple graph, $\Delta(G) \leq \chi'(G) \leq \Delta(G)+1$. Hence $s(G) \geq \Delta(G)$. Suppose if '*k*' colors are used for skew edge coloring, then there are $\binom{k+1}{2}$ unordered pairs of colors and this number must be greater than or equal to the number of edges in *G*. Let *k*(*m*) denote the smallest integer '*k*' satisfying $\binom{k+1}{2} \geq m$ where '*m*' denotes the number of edges in *G*. Thus the best lower bound for *s*(*G*) is $s(G) \geq \max\{\Delta(G), k(|E(G)|)\}$ [6]. This work mainly proves that the bound on the skew chromatic index given here is sharp for uniform theta and quasi-uniform theta graphs.

III. UNIFORM THETA GRAPHS

Definition 3.1: [10] A graph $\Theta(s_1, s_2, ..., s_n)$ which consists of two end vertices namely north pole (*N*) and south pole (*S*) joined by *n* internally disjoint paths called longitudes (*L*) of length greater than one is called a generalized theta graph, and the number of internal vertices in each longitude L_i being s_i, $1 \leq i \leq n$.

Definition 3.2: [10] A uniform theta graph $\Theta(n, l)$ is a generalized theta graph with *l* longitudes $L_1, L_2, ..., L_l$ such that $|L_1| = |L_2| = ... = |L_l| = n$, and $|L_i|$ being the number of internal vertices in L_i. See Fig. 1.

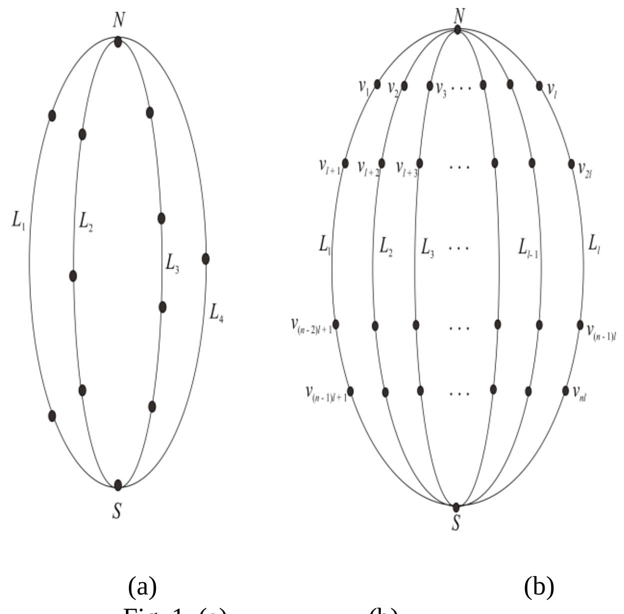


Fig. 1. (a) $\Theta(2, 3, 4, 1)$ (b) $\Theta(n, l)$

Theorem 3.1: A uniform theta graph $\Theta(n, l)$, $n \geq 3$ and $3 \leq l \leq 2n-2$ is skew edge colorable.

Proof: Consider a uniform theta graph $G = \Theta(n, l)$, $n \geq 3$ and $3 \leq l \leq 2n-2$. Let $V(G) = \{N\} \cup \{S\} \cup \{v_i, 1 \leq i \leq nl\}$ and $E(G) = \{(Nv_i) \cup (v_i v_{l+i}) \cup (v_{l+i} v_{2l+i}) \cup (v_{2l+i} v_{3l+i}) \cup \ldots \cup (v_{(n-2)l+i} v_{(n-1)l+i}) \cup (v_{(n-1)l+i} S), 1 \leq i \leq l\}$.

Here $|V(G)| = nl + 2$ and $|E(G)| = (n+1)l$.

Define the mappings $f : E(G) \rightarrow \{1, 2, 3, \ldots, k\}$ and $g : E(G) \rightarrow \{1, 2, 3, \ldots, k\}$ as follows where k is the smallest positive integer satisfying $\binom{k+1}{2} \geq (n+1)l$. i.e., $\binom{k+1}{2} \geq m$, the number of edges of the graph.

For $1 \leq i \leq l$, define

$$f(Nv_i) = i$$

$$f(v_i v_{l+i}) = \begin{cases} l+i, & l+i \leq k \\ r, & l+i \equiv r \pmod{k} \end{cases}$$

$$f(v_{l+i} v_{2l+i}) = \begin{cases} 2l+i, & 2l+i < k \\ k, & 2l+i \equiv 0 \pmod{k} \\ r, & 2l+i \equiv r \pmod{k} \end{cases}$$

$$f(v_{2l+i} v_{3l+i}) = \begin{cases} k, & 3l+i \equiv 0 \pmod{k} \\ r, & 3l+i \equiv r \pmod{k} \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$f(v_{(n-2)l+i} v_{(n-1)l+i}) = \begin{cases} k, & (n-1)l+i \equiv 0 \pmod{k} \\ r, & (n-1)l+i \equiv r \pmod{k} \end{cases}$$

$$f(v_{(n-1)l+i} S) = \begin{cases} k, & nl+i \equiv 0 \pmod{k} \\ r, & nl+i \equiv r \pmod{k} \end{cases}$$

The mapping 'f' assigns the different colors $\{1, 2, 3, \ldots, l\}$ to the l edges incident to the vertex N. The assignment of colors to any two adjacent edges on each of the longitudes L_i and to the edges incident to the vertex S is a proper coloring. Suppose $f(v_{l+i} v_{2l+i}) = f(v_{2l+i} v_{3l+i})$, then $2l + i$ and $3l + i$ are in the same residue class [r] modulo k. This is true if and only if $3l + i \equiv 2l + i \pmod{k}$. This implies $k \mid l$, a contradiction. Thus 'f' defines a proper edge coloring and is called the first component coloring of $\Theta(n, l)$.

For $1 \leq i \leq l$, define

$$g(Nv_i) = i$$

$$g(v_i v_{l+i}) = \begin{cases} l+i, & l+i \leq k \\ r+q, & l+i \equiv r \pmod{k} \end{cases}$$

$$g(v_{l+i} v_{2l+i}) = \begin{cases} 2l+i, & 2l+i \leq k \\ q-1, & 2l+i \equiv 0 \pmod{k} \\ r+q, & 2l+i \equiv r \pmod{k}, r+q \leq k \\ r+q-k, & 2l+i \equiv r \pmod{k}, r+q > k \end{cases}$$

$$g(v_{2l+i} v_{3l+i}) = \begin{cases} q-1, & 3l+i \equiv 0 \pmod{k} \\ r+q, & 3l+i \equiv r \pmod{k}, r+q \leq k \\ r+q-k, & 3l+i \equiv r \pmod{k}, r+q > k \end{cases}$$

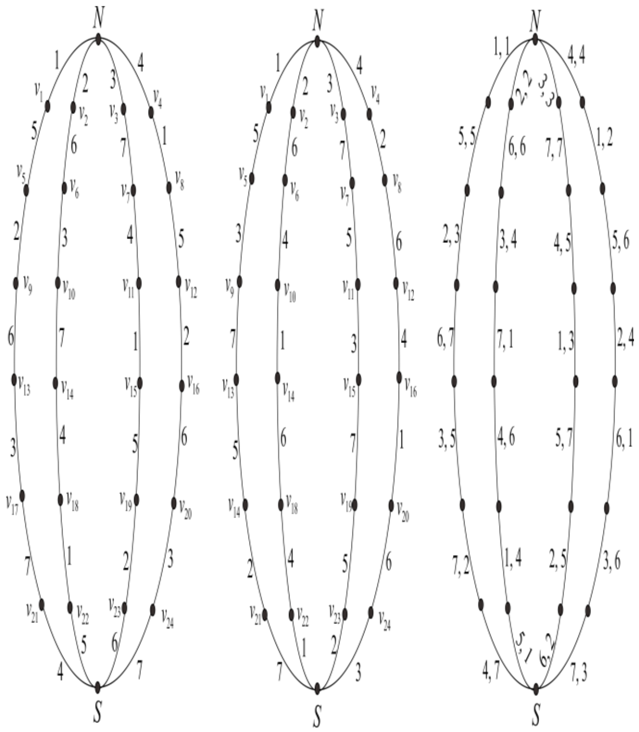
$$\vdots \qquad \vdots \qquad \vdots$$

$$g(v_{(n-2)l+i} v_{(n-1)l+i}) = \begin{cases} q-1, & (n-1)l+i \equiv 0 \pmod{k} \\ r+q, & (n-1)l+i \equiv r \pmod{k}, r+q \leq k \\ r+q-k, & (n-1)l+i \equiv r \pmod{k}, r+q > k \end{cases}$$

$$g(v_{(n-1)l+i} S) = \begin{cases} q-1, & nl+i \equiv 0 \pmod{k} \\ r+q, & nl+i \equiv r \pmod{k}, r+q \leq k \\ r+q-k, & nl+i \equiv r \pmod{k}, r+q > k \end{cases}$$

where 'q' is the quotient in each case mod k. Thus 'g' defines a proper edge coloring and is called the second component coloring of $\Theta(n, l)$. The two component colorings of $\Theta(n, l)$ defined by $f(E(G))$ and $g(E(G))$ is as follows. The first 'k' edges are assigned the colors of the form (c, c), c = 1, 2, 3, …, k. In the second set of 'k' edges, the first k – 1 edges are assigned the colors of the form (c, c + 1), c = 1, 2, 3, …, k – 1. The k^{th} edge is assigned the color (k, 1) as the ordered pair (k, k + 1) is not permissible. In the third set of 'k' edges, the first k – 2 edges are assigned the colors of the form (c, c + 2), c = 1, 2, 3, …, k – 2. The (k – 1)th and the k^{th} edge are assigned the colors (k – 1, 1) and (k, 2) respectively as the ordered pairs (k – 1, k + 1) and (k, k + 2) are not permissible. Thus the pairs of colors (f, g) assigned to each edge in $\Theta(n, l)$ are all distinct and forms the skew edge coloring of G. See Fig. 2.

Theorem 3.2: Let $G = \Theta(n, l)$, $n \geq 3$ and $3 \leq l \leq 2n-2$ be a uniform theta graph. Then $s(G) = \left\lceil \frac{-1 + \sqrt{1 + 8(n+1)l}}{2} \right\rceil$.



(a) (b) (c)

Fig. 2. (a) First component coloring of $\Theta(6, 4)$
(b) Second component coloring of $\Theta(6, 4)$
(c) Skew edge coloring of $\Theta(6, 4)$ with $s(G) = 7$

Proof: Since there are $(n+1)l$ edges in G, its skew edge coloring will require at least $(n+1)l$ pairs of colors. If 'k' colors are used for coloring, then there are $\binom{k+1}{2}$ unordered pairs. This $\binom{k+1}{2} \geq |E(G)|$. Therefore fix '$k$' in such a way that $\binom{k+1}{2} \geq (n+1)l$. It follows that $k^2 + k - (2nl + 2l) \geq 0$. This is a quadratic equation in 'k'. Its solution gives the required value of $s(G) = k = \left\lceil \frac{-1+\sqrt{1+8(n+1)l}}{2} \right\rceil$.

By comparing with the bound on $s(G)$ as discussed in section II, the skew chromatic index obtained is equivalent to $k(|E(G)|)$ which is an optimal solution for the skew chromatic index of uniform theta graphs $\Theta(n,l)$ for $n \geq 3$ and $3 \leq l \leq 2n-2$. Hence for a given $\Theta(n,l)$, we immediately obtain its $s(G)$ by a simple manipulation.

IV. QUASI UNIFORM THETA GRAPHS

In this section, we determine the skew chromatic index of quasi-uniform theta graphs for $n \geq 3$ and $3 \leq l \leq 2n-2$.

Definition 4.1: [10] A quasi-uniform theta graph is a generalized theta graph with l longitudes $L_1, L_2, \ldots, L_l$ such that $|L_1| = |L_2| = \ldots = |L_{l-1}| < |L_l|$, and $|L_i|$ being the number of internal vertices in L_i. It is said to be even or odd according as $|L_{l-1}| + |L_l|$ is even or odd. It is obvious that every uniform theta graph is also an even quasi-uniform theta graph.
See Fig. 3.

Theorem 4.1: Let G be a quasi-uniform theta graph (even or odd) with l longitudes $L_1, L_2, \ldots, L_l$ and n vertices on each $L_i, 1 \leq i \leq l-1$ and $n+t$ vertices on L_l, where $t = |L_l| - |L_{l-1}|$, $n \geq 3$ and $3 \leq l \leq 2n-2$. Then G admits skew edge coloring and its skew chromatic index is $s(G) = \left\lceil \frac{-1+\sqrt{1+8[(n+1)l+t]}}{2} \right\rceil$.

Proof: Let $G = \Theta(s_1, s_2, \ldots, s_{l-1}, s_l)$ where $s_1 = s_2 = \ldots = s_{l-1} = n$, $s_l = n+t$ be a quasi-uniform theta graph with $n \geq 3$ and $3 \leq l \leq 2n-2$.

Let $V(G) = \{N\} \cup \{S\} \cup \{v_i, 1 \leq i \leq nl\} \cup \{v_{nl+i}, 1 \leq i \leq t\}$ and $E(G) = \{(Nv_i) \cup (v_i v_{l+i}) \cup (v_{l+i} v_{2l+i}) \cup \ldots \cup (v_{(n-2)l+i} v_{(n-1)l+i}),$ $1 \leq i \leq l\} \cup \{v_{(n-1)l+i} S, 1 \leq i \leq l-1\} \cup \{v_{nl+i} v_{nl+(i+1)}, 0 \leq i \leq t-1\}$ $\cup \{v_{nl+t} S\}$.

Here $|V(G)| = nl + t + 2$ and $|E(G)| = (n+1)l + t$.

Define the mappings $f: E(G) \to \{1, 2, 3, \ldots, k\}$ and $g: E(G) \to \{1, 2, 3, \ldots, k\}$ as follows where k is the smallest positive integer satisfying $\binom{k+1}{2} \geq (n+1)l + t$.

For $1 \leq i \leq l$, define

$$f(Nv_i) = i$$

$$f(v_i v_{l+i}) = \begin{cases} l+i, & l+i \leq k \\ r, & l+i \equiv r \pmod{k} \end{cases}$$

$$f(v_{l+i} v_{2l+i}) = \begin{cases} 2l+i, & 2l+i < k \\ k, & 2l+i \equiv 0 \pmod{k} \\ r, & 2l+i \equiv r \pmod{k} \end{cases}$$

$$f(v_{2l+i} v_{3l+i}) = \begin{cases} k, & 3l+i \equiv 0 \pmod{k} \\ r, & 3l+i \equiv r \pmod{k} \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$f(v_{(n-2)l+i} v_{(n-1)l+i}) = \begin{cases} k, & (n-1)l+i \equiv 0 \pmod{k} \\ r, & (n-1)l+i \equiv r \pmod{k} \end{cases}$$

For $1 \leq i \leq l-1$,

$$f(v_{(n-1)l+i} S) = \begin{cases} k, & nl+i \equiv 0 \pmod{k} \\ r, & nl+i \equiv r \pmod{k} \end{cases}$$

For $0 \leq i \leq \left\lfloor \frac{t}{2} \right\rfloor$,

$$f(v_{nl+i} v_{nl+(i+1)}) = \begin{cases} k, & nl+l+2i \equiv 0 \pmod{k} \\ r, & nl+l+2i \equiv r \pmod{k} \end{cases}$$

$$f(v_{nl+t} S) = \begin{cases} k, & nl+l+1 \equiv 0 \pmod{k} \\ r, & nl+l+1 \equiv r \pmod{k} \end{cases}$$

The following two cases arise when we color the remaining $\left\lfloor \frac{t-1}{2} \right\rfloor$ edges $(v_{nl+\frac{t}{2}+1} v_{nl+\frac{t}{2}+2}), (v_{nl+\frac{t}{2}+2} v_{nl+\frac{t}{2}+3}), \ldots, (v_{nl+t-2} v_{nl+t-1}),$ $(v_{nl+t-1} v_{nl+t})$ on the longitude L_l.

case (i) : t even. For $1 \leq i \leq \frac{t}{2} - 1$,

$$f(v_{nl+\frac{t}{2}+i} v_{nl+\frac{t}{2}+(i+1)}) = \begin{cases} k, & nl+l+t-(2i-1) \equiv 0 \pmod{k} \\ r, & nl+l+t-(2i-1) \equiv r \pmod{k} \end{cases}$$

case (ii) : t odd. For $1 \leq i \leq \left\lfloor \frac{t}{2} \right\rfloor$,

$$f(v_{nl+\left\lfloor \frac{t}{2} \right\rfloor+i} v_{nl+\left\lfloor \frac{t}{2} \right\rfloor+(i+1)}) = \begin{cases} k, & nl+l+2\left\lfloor \frac{t}{2} \right\rfloor-(2i-1) \equiv 0 \pmod{k} \\ r, & nl+l+2\left\lfloor \frac{t}{2} \right\rfloor-(2i-1) \equiv r \pmod{k} \end{cases}$$

Thus 'f' defines a proper edge coloring and is called the first component coloring of the quasi-uniform theta graph.

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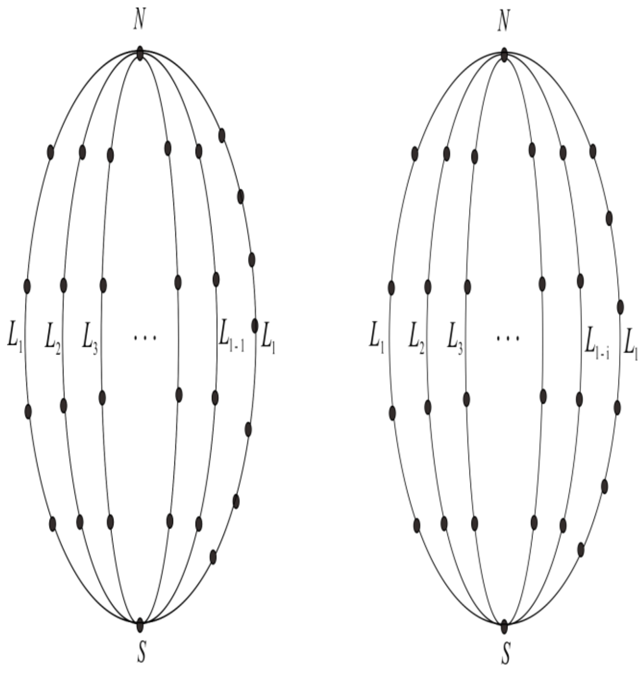

(a) (b)

Fig. 3. (a) Odd quasi-uniform theta graph
(b) Even quasi-uniform theta graph

For $1 \le i \le l$, define

$$g(Nv_i) = i$$

$$g(v_i v_{l+i}) = \begin{cases} l+i, & l+i \le k \\ r+q, & l+i \equiv r \ (\mathrm{mod}\ k) \end{cases}$$

$$g(v_{l+i} v_{2l+i}) = \begin{cases} 2l+i, & 2l+i \le k \\ q-1, & 2l+i \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & 2l+i \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & 2l+i \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

$$g(v_{2l+i} v_{3l+i}) = \begin{cases} q-1, & 3l+i \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & 3l+i \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & 3l+i \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$g(v_{(n-2)l+i} v_{(n-1)l+i}) = \begin{cases} q-1, & (n-1)l+i \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & (n-1)l+i \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & (n-1)l+i \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

For $1 \le i \le l-1$,

$$g(v_{(n-1)l+i} S) = \begin{cases} q-1, & nl+i \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & nl+i \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & nl+i \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

For $0 \le i \le \left\lfloor \frac{t}{2} \right\rfloor$,

$$g(v_{nl+i} v_{nl+(i+1)}) = \begin{cases} q-1, & nl+l+2i \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & nl+l+2i \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & nl+l+2i \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

$$g(v_{nl+t} S) = \begin{cases} q-1, & nl+l+1 \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & nl+l+1 \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & nl+l+1 \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

The following two cases arise when we color the remaining $\left\lfloor \frac{t-1}{2} \right\rfloor$ edges $(v_{nl+\frac{t}{2}+1} v_{nl+\frac{t}{2}+2}), (v_{nl+\frac{t}{2}+2} v_{nl+\frac{t}{2}+3}), \ldots, (v_{nl+t-2} v_{nl+t-1})$, $(v_{nl+t-1} v_{nl+t})$ on the longitude L_l.

case (i) : t even. For $1 \le i \le \frac{t}{2} - 1$,

$$g(v_{nl+\frac{t}{2}+i} v_{nl+\frac{t}{2}+(i+1)}) = \begin{cases} q-1, & nl+l+t-(2i-1) \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & nl+l+t-(2i-1) \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & nl+l+t-(2i-1) \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

case (ii) : t odd. For $1 \le i \le \left\lfloor \frac{t}{2} \right\rfloor$,

$$g(v_{nl+\left\lfloor \frac{t}{2} \right\rfloor+i} v_{nl+\left\lfloor \frac{t}{2} \right\rfloor+(i+1)}) = \begin{cases} q-1, & nl+l+2\left\lfloor \frac{t}{2} \right\rfloor-(2i-1) \equiv 0 \ (\mathrm{mod}\ k) \\ r+q, & nl+l+2\left\lfloor \frac{t}{2} \right\rfloor-(2i-1) \equiv r \ (\mathrm{mod}\ k),\ r+q \le k \\ r+q-k, & nl+l+2\left\lfloor \frac{t}{2} \right\rfloor-(2i-1) \equiv r \ (\mathrm{mod}\ k),\ r+q > k \end{cases}$$

where 'q' is the quotient in each case mod k. Thus 'g' defines a proper edge coloring and is called the second component coloring of the quasi-uniform theta graph. The two component colorings defined by $f(E(G))$ and $g(E(G))$ for each edge in the quasi-uniform theta graph $\Theta(s_1, s_2, \ldots, s_{l-1}, s_l)$ are all distinct and thus give the skew edge coloring of G. The first 'k' edges are assigned the colors of the form (c, c), $c = 1, 2, 3, \ldots, k$. In the second set of 'k' edges, the first $k-1$ edges are assigned the colors of the form $(c, c+1)$, $c = 1, 2, 3, \ldots, k-1$. The k^{th} edge is assigned the color $(k, 1)$ as the ordered pair $(k, k+1)$ is not permissible. In the third set of 'k' edges, the first $k-2$ edges are assigned the colors of the form $(c, c+2)$, $c = 1, 2, 3, \ldots, k-2$. The $(k-1)^{th}$ and the k^{th} edge are assigned the colors $(k-1, 1)$ and $(k, 2)$ respectively as the ordered pairs $(k-1, k+1)$ and $(k, k+2)$ are not permissible. Thus the pairs of colors (f, g) assigned to each edge in the quasi-uniform theta graph are all distinct and forms the skew edge coloring of G. See Fig. 4.

Proceeding as in *Theorem* 3.2, an optimal s(G) for quasi-uniform theta graph is obtained as

$$s(G) = \left\lceil \frac{-1+\sqrt{1+8m}}{2} \right\rceil = \left\lceil \frac{-1+\sqrt{1+8[(n+1)l+t]}}{2} \right\rceil.$$

By comparing with the bound on $s(G)$ as discussed in Section II, the skew chromatic index obtained is equivalent to $k(|E(G)|)$ which is an optimal solution for the skew chromatic index of quasi-uniform theta graphs.

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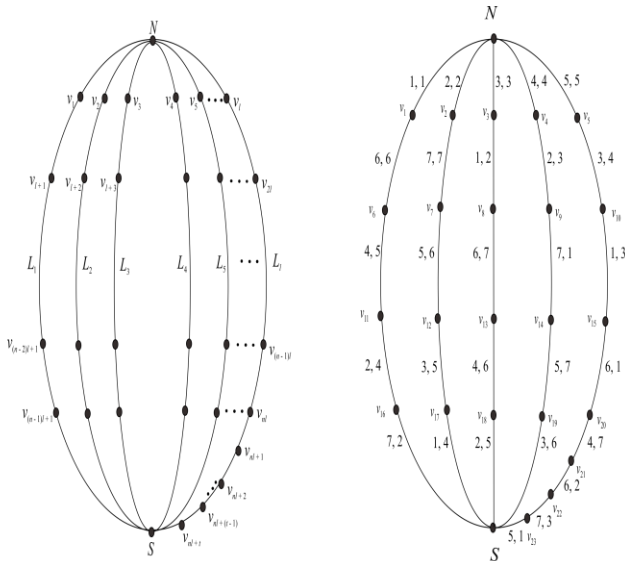

(a) (b)

Fig. 4. (a) Quasi-uniform theta graph with *l* longitudes
(b) Odd quasi-uniform with n = 4 and t = 3

V. CONCLUSION

The skew edge coloring and hence the skew chromatic index of uniform theta and quasi-uniform theta graphs is obtained. Determining the skew chromatic index of other interconnection networks is quite interesting. Finding the skew chromatic index of cyclic snakes is under consideration.

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