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Edge -Balance Index Sets of HELM

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Abstract - The edge-balance index set of a graph G (V, E) was defined by Chopra, Lee and Su[1] in 2010 as follows: For an edge labeling $f:E(G)\rightarrow\{0,1\}$, a partial vertex labeling $f^*: V(G) \rightarrow \{0, 1\}$ is defined as

$$f^*(v) = \begin{cases} 0 \text{, if more edges with label 0 are incident to v} \\ 1 \text{, if more edges with label 1 are incident to v} \\ \text{unlabeled , otherwise} \end{cases}$$

For i = 0 or 1, let $A = \{uv \in E : f(uv) = i\}$ and $B = \{v \in V : f^*(v) = i\}$

Let $e_G(i) = |A|$ and $v_G(i) = |B|$. The edge balance index set of G denoted as EBI(G) is computed as $EBI(G)=\{|v_G(0) - v_G(1)|$: the edge labeling f satisfies $|e_G(0) - e_G(1)| \leq 1\}$. The edge-balance index set for the fan graph F_{n-1} where $F_{n-1} = P_{n-1}+K_1$ and wheel graph W_n, where $W_n = C_{n-1} + K_1$ was obtained by Lee, Tao, Lo[5]. In this paper, we compute the edge-balance index set for the Helm graph, where the Helm graph is defined as the graph obtained from a wheel graph by attaching a pendant edge at each vertex of the n- cycle.

Keywords: Binary labeling, Edge- friendly labeling, Edge-balanced index set

I. INTRODUCTION

In 1967 Rosa [6] introduced the graph labeling methods called β-valuation as a tool for decomposing the complete graph into isomorphic sub graphs. Later on, this β – valuation was renamed as graceful labeling by Golomb [9]. Over the past five decades a large number of graph labeling methods have been studied on various types of graphs. In 2010 Chopra, Lee and Su [1] introduced a new graph labeling method called the edge-balanced labeling. To understand the edge-balanced labeling one must be familiar with some basic labelings like binary labeling, friendly labeling and edge-friendly labeling. The vertex labeling f^* is said to be a *binary labeling* if $f^*: V(G) \rightarrow \{0, 1\}$ such that each edge xy is assigned the label $|f^*(x) - f^*(y)|$. A binary labeling is called a *friendly labeling* if $|v_G(0) - v_G(1)| \leq 1$, where $v_G(0)$ is the number of vertices labeled 0 and $v_G(1)$ is the number of vertices labeled 1. An edge labeling f of a graph G is said to be *edge-friendly* if $|e_G(0) - e_G(1)| \leq 1$, where $e_G(0)$ is the number of edges labeled 0 and $e_G(1)$ is the number of edges labeled 1.

For an edge labeling $f : E(G) \rightarrow \{0, 1\}$, if a partial vertex labeling $f^*: V(G) \rightarrow \{0, 1\}$ is defined as follows:

$$f^*(v) = \begin{cases} 0 \text{, if more edges with label 0 are incident to v} \\ 1 \text{, if more edges with label 1 are incident to v} \\ \text{unlabeled , otherwise} \end{cases}$$

and for i = 0 or 1, let $A = \{uv \in E : f(uv) = i\}$, $B = \{v \in V : f^*(v) = i\}$ and $e_G(i) = |A|$, $v_G(i) = |B|$ then the edge labeling f is called the *edge-balanced labeling.* From the edge balanced labeling , the edge balance index set is computed. The *edge balance index set* of G denoted as EBI(G) is computed as $EBI(G) = \{|v_G(0) - v_G(1)|$: the edge labeling f satisfies $|e_G(0) - e_G(1)| \leq 1\}$. Lee *et al* [2] has obtained the edge-balance index set for the fan graph F_{n-1} where $F_{n-1} = P_{n-1} + K_1$ and wheel graph W_n, where $W_n = C_{n-1} + K_1$. Lee, Tao and Lo[6] have found out the edge-balance index sets of stars, paths and double stars. Wang, Lee, *et al* [7] and [8] have proved the edge-balance index sets of complete graph, prisms (prisms are graphs of the form $C_m \times P_n$) and Möbius ladder (the graph obtained from the ladder $P_n \times P_2$ by joining the opposite end points of the two copies of P_n).

In this paper, we compute the edge-balance index set for the Helm graph, where the Helm graph is the graph obtained from a wheel graph by attaching a pendant edge at each vertex of the n- cycle.

II. MAIN RESULT

In this section we obtain the edge-balance index set for the Helm graph H_p.

Theorem

The Edge-balance index set for the Helm graph H_p is computed as {(m)}, {(m-1)}, {0}, {2}, {(m – 2 – i)}, {(m - i)}.

Proof

Let H_p be the Helm graph with p vertices. Let q be the number of edges in the Helm graph H_p. We describe the helm H_p as follows:

Denote the central vertex of H_p as v_0. The vertex v_0 is called the hub vertex of the helm graph. Denote the vertices in the cycle of the helm as $v_1, v_2, v_3,\ldots, v_{m-1}, v_m$ in the clockwise direction. Denote the end-vertices of the helm as $v_{m+1}, v_{m+2}, v_{m+3},\ldots, v_{2m-1}, v_{2m}$ in the clockwise direction. (See Figure 1). Note that the degree of v_0 is m. Also note that $p = 2m + 1$. Denote the edges incident with v_0 as $d_1, d_2, d_3,\ldots, d_m$ in the clockwise direction. Denote the edges of the cycle in the helm as $e_1, e_2, e_3,\ldots, e_m$ in the clockwise direction. Denote the pendant edges of the helm as $c_1, c_2, c_3,\ldots, c_m$ again in the clockwise direction. Note that q = 3m.

In order to compute the edge-balance index set for the Helm graph, we first label the edges of the Helm H_p and verify that these edge labels are edge-friendly as follows. Denote the number of edges that are labeled 0 in H_p as $e_{Hp}(0)$ and the number of edges that are labeled 1 in H_p as $e_{Hp}(1)$. If $|e_{Hp}(0) - e_{Hp}(1)| \leq 1$, then the edge labels of the Helm H_p are edge-friendly.Now we label the vertices v_i of H_p as follows. Let $e^{vi}(0)$ denote the number of edges with label 0 incident with the vertex v_i. Let $e^{vi}(1)$ denote the number of edges with label 1 incident with the vertex v_i.

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Let $f^*(v_i) = \begin{cases} \max(e^{vi}(0), e^{vi}(1)) \\ \text{unlabeled, if } e^{vi}(0) = e^{vi} \end{cases}$ (1)

Now we calculate the edge balance index set of the helm H_p as follows. Let $v_{Hp}(0)$ be the number of vertices that are labeled 0 in H_p and let $v_{Hp}(1)$ be the number of vertices that are labeled 1 in H_p. Let $v_{Hp}(x)$ be the number of vertices that are unlabeled in H_p. We obtain the edge-balance index sets for helm as $EBI(H_p) = \{| v_{Hp}(0) - v_{Hp}(1)|\}$.

There are totally three different ways or methods for labeling the edges of the helm in order to obtain the edge-balance index set for the helm graph H_p. We discuss only one method in the proof of this theorem as follows.

Case (i) : When m is odd

In this case we take m = 2i + 1, i = 1, 2, 3, ...
Define the edge labels of helm H_p as follows.

$f(e_{2i+1}) = 0$, $0 \leq i \leq \lfloor m/2 \rfloor$
$f(e_{2i}) = 1$, $1 \leq i \leq \lfloor m/2 \rfloor$
$f(d_i) = 1$, $1 \leq i \leq m$
$f(c_i) = 0$, $1 \leq i \leq m$ (2)

From the equation (2), the number of edges with label 0 is calculated as

$e_{Hp}(0) = \lfloor q/2 \rfloor + 1$ and (3)

the number of edges with label 1 is calculated as

$e_{Hp}(1) = \lfloor q/2 \rfloor$ (4)

Hence $|e_{Hp}(0) - e_{Hp}(1)| = |\lfloor q/2 \rfloor + 1 - \lfloor q/2 \rfloor| = 1.$ (5)

Therefore, the edge labels of H_p are edge-friendly.

From the equation (1), the number of vertices with label 0 is calculated as

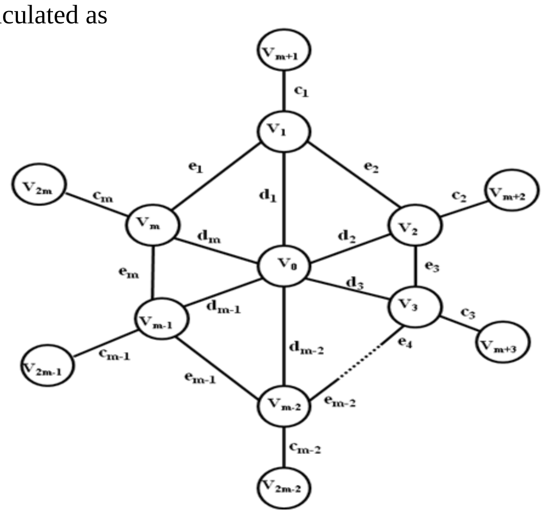

Figure 1. Helm H_p

$v_{Hp}(0) = m + 1$ and (6)

the number of vertices with label 1 is calculated as

$v_{Hp}(1) = 1$ (7)

Therefore, the edge-balance index set of H_p is

$EBI(H_p) = \{| v_{Hp}(0) - v_{Hp}(1)|\}$
$= \{|m + 1 - 1|\}$
$EBI(H_p) = \{m\}$ (8)

Case(ii) : When m is even

In this case we take m = 2i, i = 2, 3, ...
Define the edge labels of helm H_p as follows.

$f(e_{2i+1}) = 0$, $0 \leq i \leq \lfloor m/2 \rfloor - 1$
$f(e_{2i}) = 1$, $1 \leq i \leq \lfloor m/2 \rfloor$
$f(d_i) = 1$, $1 \leq i \leq m$
$f(c_i) = 0$, $1 \leq i \leq m$ (9)

From the equation (9), the number of edges with label 0 is calculated as

$e_{Hp}(0) = q/2$ and (10)

the number of edges with label 1 is calculated as

$e_{Hp}(1) = q/2$ (11)

Hence $| e_{Hp}(0) - e_{Hp}(1) | = | q/2 - q/2 | = 0$ (12)

Therefore, the edge labels of H_p are edge-friendly.

From the equation (1), the number of vertices with label 0 is calculated as

$v_{Hp}(0) = m$ and (13)

the number of vertices with label 1 is calculated as

$v_{Hp}(1) = 1$ (14)

Therefore, the edge-balance index set of H_p is

$EBI(H_p) = \{| v_{Hp}(0) - v_{Hp}(1)|\}$
$= \{|m - 1|\}$
$EBI(H_p) = \{m - 1\}$ (15)

III. CONCLUSION

By the first method, the edge balance index set of Helm H_p is computed as

$$EBI(H_p) = \begin{cases} \{m\}, \text{ when m is odd} \\ \{m-1\}, \text{ when m is even} \end{cases}$$

In a similar manner we can compute the edge-balance index set of H_p by the other two methods. By the second method, the edge balance index set of Helm H_p is obtained as

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$EBI(H_p) =$ {2} , when m is odd
{0} , when m is even

By the third method, the edge balance index set of Helm H_p is obtained as

$EBI(H_p) =$ {(m – 2 – i)} , when m is odd
{(m – i)} , when m is even

Hence the Edge-balance index set for the Helm graph H_p is computed as {(m)}, {(m-1)}, {0}, {2}, {(m – 2 – i)}, {(m - i)}. The illustrations for the cases (i) & (ii) of the first method in the proof of the above theorem is given in the appendix.

Applications: One interesting application of the edge-balanced labeling is playing the e*dge-balanced game* on any graph. The edge-balanced game is explained as follows: Player A and B take turns assigning labels 0 and 1, respectively, to the edges of a graph until all the edges have been labeled. No edge can be labeled twice. Given any vertex, if the number of 0-edges emanating from that vertex is greater than the number of 1-edges, then that vertex belongs to A. If the number of 0-edges is less than the number of 1-edges, the vertex belongs to B. If the number of 0-edges is equal to the number of 1-edges, the vertex belongs to neither player. The winner is the player who claims the most vertices. Playing the edge-balanced game is more complicated when the graph is complex involving more edges incident to the vertex. Hence finding the edge-balanced index for a structurally larger graph becomes complex.

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APPENDIX

Case(i) :When m is odd : In this case we take m=2i+1. When i=11

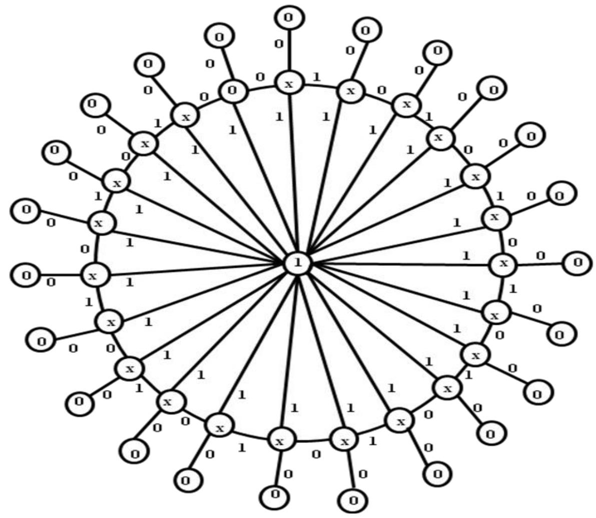

Figure 2. Edge-balance index set of H_{47}

|e(0) – e(1)| = |35 - 34| = 1 $EBI(H_{47})$ = {|v(0) – v(1)|} = {24 – 1} = {23}

Case (ii) : When m is even In this case we take m = 2i, When i = 10, m = 20

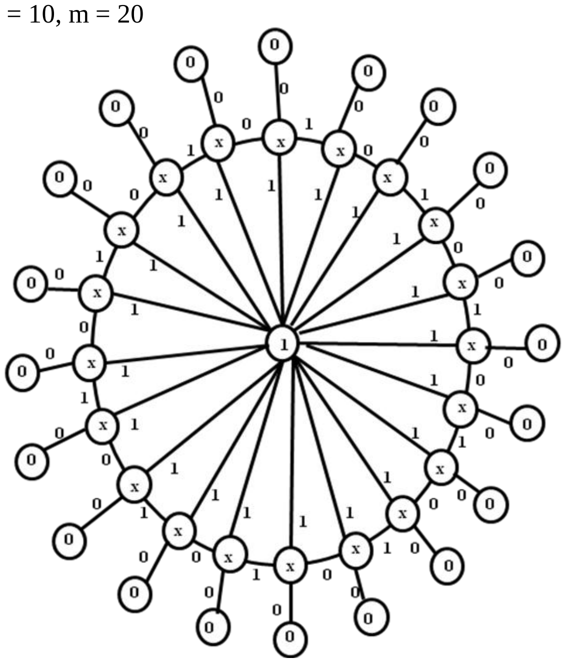

Figure 3. Edge-balance index set of H_{41}

|e(0) – e(1)| = |30 - 30| = 0
$EBI(H_{41})$ = {|v(0) – v(1)|} = {20 - 1} = {19}