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Cyclic Resolving Number of Grid and Augmented Grid Graphs

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Abstract - For an ordered set W = {w_1, w_2 … w_k} ⊆ V (G) of vertices, we refer to the ordered k-tuple r(v | W) = (d(v, w_1), d(v, w_2) … d(v, w_k)) as the (metric) representation of v with respect to W. A set W of a connected graph G is called a resolving set of G if distinct vertices of G have distinct representations with respect to W. A resolving set with minimum cardinality is called a minimum resolving set or a basis. The dimension, dim(G), is the number of vertices in a basis for G. By imposing additional constraints on the resolving set, many resolving parameters are formed. In this paper, we introduce cyclic resolving set and find the cyclic resolving number for a grid graph and augmented grid graph.

Keywords: Resolving set, Cyclic resolving number, Grid graph, distance

I. INTRODUCTION

If G is a connected graph, then the distance $d(u; v)$ between two vertices $u; v \in V(G)$ is the shortest u - v path. For an ordered set$W = \{w_1, w_2 \dots w_k\} \subseteq V(G)$ and a vertex v of G, the k-vector $r(v|W) = (d(v,w_1), d(v,w_2) \dots d(v,w_k))$ is referred as the representation of v with respect to W. If distinct vertices of G have distinct representations with respect to W, then W is called a *resolving set* for G. A resolving set containing a minimum number of vertices is called a *minimum resolving set* or a *basis* for G. The dimension, $dim(G)$, is the number of vertices in a basis for G [17].

The i-th component of $r(v|W)$ is 0 if and only if $v = w_i$, for an ordered set $W = \{w_1, w_2 \dots w_k\}$ of vertices. Hence it is enough to verify that $r(x|W) \neq r(y|W)$ for each pair of distinct vertices $x, y \in V(G) \setminus W$, to show that W is a resolving set. Slater [19, 20] introduced this concept using locating set for a resolving set and referred to the cardinality of a minimum resolving set in a graph G as its location number. Independently, Harary and Melter [7] discovered these concepts but used the term metric dimension for a location number. These concepts have also been investigated by Johnson [8] of the Pharmaceutical Company while attempting to develop a capability of large data sets of chemical graphs and he also noted that the problem of finding the metric dimension is NP-hard. Independently, Chartrand et al. [4] have also discovered the concept of a resolving set and a minimum resolving set.

Resolving sets have applications in chemistry for representing chemical compounds [8],problems of network discovery and verification [3, 21],pattern recognition and image processing which involve the use of hierarchical data structures [11], robot navigation [9] and in areas like coin weighing problems [6].The problem of computing the metric dimension of a graph is *NP*-complete [5].This problem remains *NP*-complete for bipartite graphs proved by Manuel et al. [14]. The metric dimension problem has also been studied for trees, multi-dimensional grids [9], torus networks [13], Benes networks [14], honeycomb networks [15], and Illiac networks [16]. For graphs modeled by an interconnection networks, a resolving set represents a set of detecting devices in a network so that for every station in the network, there are two detecting devices whose distances from the station are distinct. This is possible as there is a distance between each vertex in a minimum resolving set. To have an easy access between the devices, the distance between devices should be small [18]. This lead the introduction of connected resolving sets and was introduced by Saenopholphat et al. [17].Many resolving parameters like connected resolving parameter, path resolving and one-factor resolving set [1], star resolving set [2] have been analyzed. This paper reports the resolving parameter of a graph G when the resolving set induces a cycle.

II. CYCLIC RESOLVING NUMBER

A resolving set W of a graph G is said to be *connected* if the subgraph $G[W]$ induced by W is a nontrivial connected subgraph of G. The *connected resolving number*$cr(G)$ is the minimum cardinality of a connected resolving set W in G. A connected resolving set of cardinality $cr(G)$ is called*cr-set* of G. Clearly, every connected resolving set is a resolving set. It was noted in [17] that if G is a connected graph of order $n \geq 3$, then $1 \leq dim(G) \leq cr(G) \leq n - 1$. Further, $dim(G) = cr(G)$ if and only if G contains a connected basis. For a connected graph G of order $n \geq 2$, $cr(G) = 1$ if and only if $G = P_n$,$cr(G) = n - 1$ if and only if $G = K_n$ or $G = K_{1, n-1}$. Moreover, for each pair k, n with $1 \leq k \leq n - 1$, there is a connected graph of order n with connected resolving number k[17].

If the subgraph$G[W]$ induced by a resolving set W of G is a cycle, then $G[W]$ is a *cyclic resolving set*. The minimum cardinality of a cyclic resolving set in a graph G is called the *cyclic resolving number* of G, denoted by $cyr(G)$. In other words, for minimum positive integers m, n such that $G[W] \cong mC_n$, then $m$is a*cyclic resolving number*.

III. GRID NETWORKS

An interconnection network is modeled as an undirected graph $G = (V, E)$ with vertex set$V$ and edge set E where there are no multiple edges or self-loops. The vertices of a graph represent the nodes (processing elements, memory models or switches) of the network and the edges correspond to communication lines. Three basic attributes such as degree, diameter and node disjoint paths are necessary for developing an interconnection network. In addition to this, optimal algorithms for various nodes of packet communication, embeddability, symmetry properties and recursive scalability are the complex attributes to be considered. A few examples of interconnection networks are tree, grid (especially the 2-dimensional grid $M_{n\times n}$), hypercube, *k*-ary*n*-cube, OTIS-Network and WK recursive grid. The advantage of grid network is that it adapts dynamically to the changes in the structure of the network [10].

A *two-dimensional grid graph* [12], denoted by $G(m,n)$, is the graph Cartesian product $P_m \times P_n$ of path graphs on m and n vertices. Thevertex set and edge set of an $m\times n$ Grid Graph $G(m, n)$ are:$V = \{(i, j): 0 \leq i < m, 0 \leq j < n\}$ and$E = \{(i,j),(i',j'): |i - i'| + |j - j'| = 1\}$.Hence the number of vertices and edges of a grid graph $G(m, n)$ are mn and $2mn - m - n$.The grid graphs are *bipartite* graphs and Hamiltonian if either the number of rows or columns is even. In this paper, we denote the vertices of an $m\times n$ Grid Graph $G(m, n)$ as (i, j) where i denotes the row and j denotes the column.

IV. CYCLIC RESOLVING NUMBER OF GRID NETWORKS

Lemma4.1: Let G denote the $m\times n$ grid graph. Then $cyr(G)>1$, where $m\geq 3; n\geq 4$.

Proof:
Consider the vertices (i, j), $(i + 1, j)$, $(i + 1, j + 1)$, $(i, j + 1)$, $0 \leq i \leq m - 1$, $0 \leq j \leq n - 1$ which induces a 4-cycle. i. e., $C = \{(i, j), (i + 1, j), (i + 1, j + 1), (i, j + 1)\}$, $0 \leq i \leq m - 1$, $0 \leq j \leq n - 1$.
To prove that C does not resolve G.
For $0 \leq i<m - 1$, $0 \leq j<n - 1$, $r((i, j + 2) \mid C) = r((i + 2, j + 1) \mid C)$ or in other words, the vertices $(i, j + 2)$ and $(i + 2, j + 1)$ have the representation with respect to C. If $i = m - 1$, $0 \leq j<n - 1$, $r((m - 1, j + 2) \mid C) = r((m - 3, j + 1) \mid C)$, for $0 \leq i<m - 1$, $j = n - 1$, $r((i, n - 3) \mid C) = r((i + 2, n - 2) \mid C)$ and for $i = m - 1$, $j = n - 1$, $r((m - 1, n - 3) \mid C) = r((m - 3, n - 2) \mid C)$. Hence $cyr(G)>1$.

Theorem4.1: For an $m\times n$ grid graph G. Then $cyr(G)=2$,$m\geq 3$and$n \geq 4$.

Proof: By Lemma4. 1,$cyr(G) > 1$. Let $S= C_4^1 \cup C_4^2$ where C_4^1= {(0, 0), (0, 1), (1, 1), (1, 0)} and C_4^2 = {(0, $n - 2$), (0, $n - 1$), (1, $n - 1$), (1, $n - 2$)}.

We claim that S is a cyclic resolving set.

For any two vertices in the same row or column, $d((0, 0), (i, j)) \neq d((0, 0), (i, j + 1))$, $0 \leq i<m$, $0 \leq j<n$ or $d((0, 0), (i, j)) \neq d((0, 0), (i + 1, j))$, $0 \leq i<m$, $0 \leq j<n$.

Consider any two vertices in different rows or columns. Then $d(x, (i + 2, j + 2)) = d(x, (i + 1, j + 1))$ for $x\in C_4^1$, $0 \leq i<m - 1$, $0 \leq j<n - 1$. Similarly, $d(x, (m - 1, j + 2)) = d(x, (m - 3, j + 1))$ for $i = m - 1$, $0 \leq j<n - 1$, $d(x, (i, n - 3)) = d(x, (i + 2, n - 2))$ and $i = m - 1$, $j = n - 1$, $d(x, (m - 1, n - 3)) = d(x, ((m - 3, n - 2))$. Pair of vertices with same representation with respect to C_4^1 is resolved by (0, $n - 1$). Hence S resolves all pairs of vertices in G.
By the structure of an $m\times n$ grid graph G, there exists cycles of length $2l + 2$, $1 \leq l \leq m + n - 2$. This implies that C_4 is a cycle of minimum length. Since the set S which induces $2C_4$ is cyclic resolving set, $cyr(G)=2$.

Figure 1 is a 5 ×7 Grid Graph$G$ and the cyclic resolving sets of G are{(0, 0), (0, 1), (1, 1), (1, 0)}, {(0, 5), (0, 6), (1, 6), (1, 5)}.

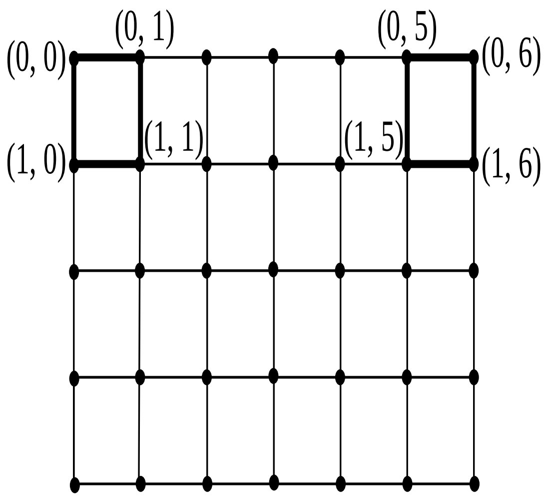

Figure 1: 5 ×7 Grid Graph with its Cyclic Resolving Set

Corollary4.1:For an $m\times n$ grid graph G, $cyr(G)=1$ if $m = 2$ and $n\geq 3$ or $m\geq 3$ and $n = 2$.

Corollary 4. 2: For $m\geq 3$ and $n = 3$, then $cyr(G) = 2$, where G is an $m\times n$ grid graph.

V. AUGMENTED GRIDGRAPHAM(M,N)

An augmented grid $AM(m,n)$is a grid $G(m,n)$with additional edges are obtained by joining the vertices $(i + 1,j)$ and $(i, j+1)$,$0 \leq i\leq m - 1$, $0 \leq j \leq n - 1$.

Lemma5. 1: Let G be $AM(m,n)$where $m \geq 3, n \geq 4$. Then $cyr(G) >1$.

Proof:
To show that a cycle of length 3 does not resolve G.
Let $C = \{(i, j), (i, j + 1), (i + 1, j)\}$, $0 \leq i \leq m - 1$, $0 \leq j \leq n - 1$.
Clearly C induces a cycle of length 3.

For $0 \leq i<m - 1$, $0 \leq j <n - 1$, $r((i + 2, j + 1) \mid C) = r((i + 1, j + 2) \mid C)$. Similarly, if $i = m - 1$, $0 \leq j <n - 1$, $r((m - 2, j + 2) \mid C) = r((m - 3, j + 2) \mid C)$, if $0 \leq i<m - 1$, $j = n - 1$, $r((i + 2, n - 3) \mid C) = r((i + 1, n - 2) \mid C)$ and if $i = m - 1$, $j = n - 1$ $r((m - 2, n - 3) \mid C) = r((m - 3, n - 2) \mid C)$. Thus $cyr(G)> 1$.

Theorem 5. 1:$cyr(AM(m,n)) = 2$, $m\geq 4$.

Proof:
In view of Lemma 5. 1, $cyr(AM(m,n)) > 1$. Assume that$S = C_3^1 \cup C_3^2$ where C_3^1= {(0, 0), (0, 1), (1, 0)} and C_3^2 = {(0,$m - 1$) (0, $m - 2$) (1, $m - 1$)}.Here $C_3^1$and C_3^2 are cycles of length 3. Any two distinct vertices in the same row or column have distinct representation with respect to C_3^1. But $d(x, (i, j)) = d(x, (l, i - (l - 1))$ for $x\in C_3^1$, $2 \leq i<m$, $j = 1$, $1 \leq l<i$and $d(x, (i, j)) = d(x, (l, m - l +i))$ for $2 \leq i<m - 1$, $j = m - 1$, $i + 1 \leq l \leq m$.

Now pairs of vertices having the same representation are resolved by any vertex of C_3^2. Since C_3 is a cycle of minimum length and S induces $2C_3$, $cyr(AM(m, n)) = 2$.

An augmented grid graph $AM(6, 6)$ with its cyclic resolving set is depicted in Figure 2.

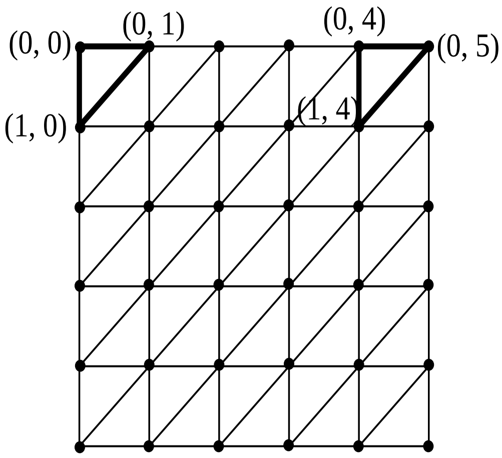

Figure 2: Cyclic Resolving Set in *AM*(6, 6)

Theorem 5. 2:$cyr(AM(m, n))=2$, $m\geq 3$, $n\geq 4$.

Proof of Theorem 5. 2 is similar to that of Theorem 5 1.

Corollary 5. 1:For$m = 2$, $n \geq 2$ and $n = 2$, $m \geq 2$, $cyr(AM(m,n)) = 1$.

Corollary 5. 2: $cyr(AM(m,n)) = 2$, whenever $n = 3$, $m \geq 3$.

VI. CONCLUSION

The cyclic resolving number has been introduced in this paper and the cyclic resolving number of Grid graph and Augmentedgrid graph has been determined. The cyclic resolving number for other interconnection networks such as Torus, Butterfly, Hypercube derived networks are under investigation.

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