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A Strategic Topsis Algorithm with Correlation Coefficient of Interval Vague Sets

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Abstract - This paper aims to develop a new method based on the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to solve Multiple Attribute Decision Making(MADM) problems for Interval Vague Sets(IVSs). A TOPSIS algorithm is constructed on the basis of the concepts of the relative-closeness coefficient computed from the correlation coefficient of IVSs. This novel method also identifiesthe positive and negative ideal solutions using the correlation coefficient of IVSs. A numerical illustration explains the proposed algorithms and comparisons are made with various existing methods.

Key-words: MADM, TOPSIS, vague sets, Interval vague sets, correlation coefficient of interval vague sets.

I. INTRODUCTION

Correlation coefficient of Fuzzy sets, Interval-valued Fuzzy sets, Intuitionistic Fuzzy sets and Interval-valued Intuitionistic Fuzzy sets are already in the literature. Various attempts are made by researchers in the recent days in defining the correlation coefficient of Intuitionistic Fuzzy sets and Interval-valued Intuitionistic Fuzzy sets. Bustince&Burillo (1995) and Hong (1998) have focussed on the correlation degree of interval valued intuitionistic fuzzy sets. Park et al. (2009) have also worked on the correlation coefficient of interval valued intuitionistic fuzzy sets and applied in multiple-attribute group decision making problems. Robinson & Amirtharaj, (2011a; 2011b; 2012a; 2012b) defined the correlation coefficient of vague sets, interval vague sets which is utilized in this work and also defined the correlation coefficient of some higher order intuitionistic fuzzy sets. Of the numerous approaches available for Decision Support Systems (DSS), one most prevalent is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), first developed by Hwang &Yoon, (1981).TOPSIS is a logical decision-making approach,dealing with the problem of choosing a solution from a set of candidate alternatives characterized in terms ofsome attributes.

The merit of the TOPSIS method suggested by Hwang &Yoon, (1981) is that it deals with both quantitative and qualitative assessments in the process evaluation with less computation. It is based upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest from the negative ideal solution. In the TOPSIS process, the performance ratings and the weights of the criteria are given as crisp values. In fuzzy TOPSIS, attribute values are represented by fuzzy numbers.Janic, (2003) stated that the TOPSIS method embraces seven steps which are:

i) Construction of normalized decision matrix;
ii) Construction of weighted-normalized decision matrix;
iii) Determining positive ideal and negative ideal solution;
iv) Calculating the separation measure of each alternative from the ideal one;
v) Calculating the relative distance of each alternative to the ideal and negative ideal solution;
vi) Ranking alternatives in descending order with respect to relative distance to idealsolution;
vii) Identifying the preferable alternative as the closest to the ideal solution.

Liu, et al, (2012) presented novel method for MCDM problems based on interval valued intuitionistic fuzzy sets(IVIFSs). Li, (2010) presented a TOPSIS-Based Nonlinear-Programming Methodology for Multi- attribute Decision Making with interval-valued intuitionistic fuzzy sets. Li& Nan, (2011) extended the TOPSIS method for Multi-attribute group decision making under Intuitionistic Fuzzy Sets (IFS) environments. Cui & Yong, (2009) developed a Fuzzy Multi-Attribute Decision Making model based on Degree of Grey Incidence and TOPSIS in the Open Tender of International Project about Contractor Prequalification Evaluation Process. Shih et al., (2001; 2007) worked on Group Decision Making for TOPSIS and its extension. In many applications, ranking of IVSs and IVIFSs plays a very important role in the decision making processes. Liu, (2009) presented a novel method of TOPSIS using a new type of score and precise function for choosing positive and negative ideal solutions in contrast to the score and accuracy functionsdefined by Chen & Tan, (1994), Hong & Choi, (2000), Wang et al., (2006) and Xu, (2007). However, Nayagam et al, (2011) proved the insufficiency of many of the score functions proposed in literature, and proposed a novel method of accuracy function for MCDM problems under IVIFS environment. In most of the previous TOPSIS techniques presented in literature, different forms of score and accuracy functions were used to identify positive and negative ideal solutions. In this work, a novel method is presented where the correlation coefficient of IVSs is used to identify positive and negative ideal solutions and for ranking alternatives based on the closeness coefficient. Comparison is made between the proposed TOPSIS and existing TOPSIS methods and some ranking functions proposed by Chen & Tan, (1994),Xu, (2007), Hong & Choi, (2000) and Liu, (2009).

Vague Set : A vague set A in a universe of discourse U is characterized by a truth membership function, t_A, and a false membership function, f_A, as follows:

$t_A : U \to [0,1]$, $f_A : U \to [0,1]$ and $t_A(u) + f_A(u) \le 1$, where $t_A(u)$ is a lower bound on the grade of membership of u derived from the evidence for u, and $f_A(u)$ is a lower bound on the grade of membership of the negation of *u* derived from the evidence against *u*.Suppose $U = \{u_1,u_2,...,u_n\}$. A vague set *A* of the universe of discourse *U* can be represented as:

$A=\sum_{i=1}^{n}[t(u_i),\ 1-f(u_i)]/u_i,\quad 0\le t(u_i)\le 1-f(u_i)\le 1,\quad i=1,2,...,n$ In other words, the grade of membership of u_i is bound to a subinterval $\left[t_A(u_i),\ 1-f_A(u_i)\right]$ of [0,1].

Let $X = \{x_1, x_2, ...,x_n\}$ be the finite universal set, VS(*X*) be the collection of vague sets and *A*, *B*∈ VS(*X*) be given by

$$A=\left\{\left\langle x,\left[t_A(x),1-f_A(x)\right]\right\rangle\ /\ x\in X\right\},$$

$$B=\left\{\left\langle x,\left[t_B(x),1-f_B(x)\right]\right\rangle\ /\ x\in X\right\}.$$

And the length of the vague values are given by $\pi_A(x) = 1 - t_A(x) - f_A(x)$, $\pi_B(x) = 1 - t_B(x) - f_B(x)$.

Interval Vague Set:

Because of the uncertainty and complexity of the decision, the values of $t_A(x)$ and $f_A(x)$ are difficult to express by exact real number values. The interval values are more flexible than the real number values and extending $t_A(x)$ and $f_A(x)$ from real number values to an interval value, an interval vague set is obtained. Obviously this set is much stronger to express uncertain data or vague data. The interval vague value is denoted as $x=<t_x, f_x>$, where $t_x=[t_x^-,t_x^+]\subseteq[0,1]$,

$f_x=[f_x^-, f_x^+]\subseteq[0,1]$, $t_x^+ + f_x^+\le 1$ and also the following equation is satisfied:

$\pi_A(x)=[1,1]-\tilde{t}_A(x)-f_A(x)=[1-t_A^+(x)-f_A^+(x),1-t_A^-(x)-f_A^-(x)]$

Operations of Interval Vague Sets:

Some basic operations of interval vague sets were discussed by Gau & Buehrer, (1994) and Li & Rao, (2001). Consider the following two interval vague values:

$$x=\left\langle\tilde{t}_x, f_x\right\rangle=\left\langle[t_x^-,t_x^+],[f_x^-, f_x^+]\right\rangle,$$

$$y=\left\langle\tilde{t}_y, f_y\right\rangle=\left\langle[t_y^-,t_y^+],[f_y^-, f_y^+]\right\rangle$$

where $\tilde{t}_x, f_x,\tilde{t}_y, f_y\subseteq[0,1]$ and $t_x^+ + f_x^+\le 1,\quad t_y^+ + f_y^+\le 1.$

The following operational rules and relations can be observed for an interval vague set:

$$\bar{x}=\left\langle f_x,\tilde{t}_x\right\rangle=\left\langle[f_x^-, f_x^+],[t_x^-,t_x^+]\right\rangle$$

$$x+y=\left\langle\tilde{t}_x+\tilde{t}_y-\tilde{t}_x\tilde{t}_y, f_x f_y\right\rangle=\left\langle[t_x^-+t_y^--t_x^-t_y^-,t_x^++t_y^+-t_x^+t_y^+],[f_x^-f_y^-, f_x^+f_y^+]\right\rangle$$

$$x\times y=\left\langle\tilde{t}_x\tilde{t}_y, f_x+f_y-f_x f_y\right\rangle=\left\langle[t_x^-t_y^-,t_x^+t_y^+],[f_x^-+f_y^--f_x^-f_y^-, f_x^++f_y^+-f_x^+f_y^+]\right\rangle$$

$$\lambda\times x=\left\langle[1-(1-t_x^-)^\lambda,1-(1-t_x^+)^\lambda],[(f_x^-)^\lambda,(f_x^+)^\lambda]\right\rangle,\lambda\ge 0.$$

The resultant of all the above operations is interval vague values. According to the operational rules, the following relations are observed:

i) $x+y=y+x$

ii) $x\times y=y\times x$

iii) $\lambda(x+y)=\lambda x+\lambda y$

iv) $\lambda_1 x+\lambda_2 x=(\lambda_1+\lambda_2)x,\quad \lambda_1,\lambda_2\ge 0.$

Correlation Coefficient Of Interval Vague Sets:

Robinson & Amirtharaj, (2012a) defined a new method for computing the correlation coefficient for Interval Vague Sets (IVSs) lying in the interval [0,1], and a new type of correlation coefficient for IVSs using α-cuts and statistical confidence intervals. The correlation coefficient of IVSs is given as follows:

Suppose *X* is a domain of *n* elements, *A* and *B* are interval vague sets,

$$A=\left\{\left\langle\left[t_A^-(x), t_A^+(x)\right],\left[f_A^-(x), f_A^+(x)\right]\right\rangle / x\in X\right\}, B=\left\{\left\langle\left[t_B^-(x), t_B^+(x)\right],\left[f_B^-(x), f_B^+(x)\right]\right\rangle / x\in X\right\}$$

and the vague degrees are given by:

$$\pi_A^-(x)=1-t_A^+(x)-f_A^+(x),\ \pi_A^+(x)=1-t_A^-(x)-f_A^-(x),$$

$$\pi_B^-(x)=1-t_B^+(x)-f_B^+(x),\ \pi_B^+(x)=1-t_B^-(x)-f_B^-(x).$$

These measures are also called hesitation degree or uncertain degree or the length of the vague value. Let IVS(*X*) be the set of all interval vague sets.

For each *A*∈ IVS(*X*), the informational vague energy of *A* is defined as follows:

$$E_{IVS}(A)=\frac{1}{2}\sum_{i=1}^{n}\left\{\left(t_A^-(x_i)\right)^2+\left(t_A^+(x_i)\right)^2+\left(1-f_A^-(x_i)\right)^2+\left(1-f_A^+(x_i)\right)^2+\left(\pi_A^-(x_i)\right)^2+\left(\pi_A^+(x_i)\right)^2\right\} \quad (1)$$

And for each *B*∈ IVS(*X*), the informational vague energy of *B* is defined as follows:

$$E_{IVS}(B)=\frac{1}{2}\sum_{i=1}^{n}\left\{\left(t_B^-(x_i)\right)^2+\left(t_B^+(x_i)\right)^2+\left(1-f_B^-(x_i)\right)^2+\left(1-f_B^+(x_i)\right)^2+\left(\pi_B^-(x_i)\right)^2+\left(\pi_B^+(x_i)\right)^2\right\} \quad (2)$$

The correlation of *A* and *B* is defined as follows:

$$C_{IVS}(A,B)=\frac{1}{2}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(t_A^-(x_i).t_B^-(x_i)\right)+\left(t_A^+(x_i).t_B^+(x_i)\right)+\left(1-f_A^-(x_i)\right)\left(1-f_B^-(x_i)\right)+\\\left(1-f_A^+(x_i)\right)\left(1-f_B^+(x_i)\right)+\left(\pi_A^-(x_i).\pi_B^-(x_i)\right)+\left(\pi_A^+(x_i).\pi_B^+(x_i)\right)\end{array}\right\} \quad (3)$$

Furthermore, the correlation coefficient of *A* and *B* is defined by the relation:

$$K_{IVS}(A,\ B)=\frac{C_{IVS}(A,\ B)}{\sqrt{E_{IVS}(A)\ .\ E_{IVS}(B)}} \quad (4)$$

Theorem1: (Robinson & Amirtharaj, 2012a)

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For all A, $B\in$ IVS(X), the correlation coefficient of IVSs satisfies:

(i) $K_{IVS}(A, B) = K_{IVS}(B, A)$.
(ii) $0 \le K_{IVS}(A, B) \le 1$.
(iii) $A = B$ iff $K_{IVS}(A, B) = 1$.

Topsis Algorithm for Interval Vague Sets:

In this paper, TOPSIS is used to confirm the order of the evaluation objects with regard to the positive and negative ideal solutions of the multi-attribute problems. A novel TOPSIS algorithm is presented where correlation coefficient is utilized to identify the positive and negative ideal solutions as well as ranking of the best alternatives. In most of the previous TOPSIS works in literature, different forms of distance and similarity functions are used to calculate the closeness coefficient. If near things are related, then distant things, although less related, are related too and in different ways reflecting their integration versus segregation in the data analysis process. Using correlation coefficient is advantageous than using any distance or similarity function because, correlation coefficient preserves the linear relationship between the variables under study. In the TOPSIS model of Liu, (2009) score function was used to identify positive and negative ideal solutions. In the proposed TOPSIS algorithm, correlation coefficient of IVSs is utilized instead of score and accuracy functions to identify the positive and negative ideal solutions.

Table-1:The decision factors involved in the proposed TOPSIS method

DECISION FACTORS	FORMULATION
Decision Alternatives	$A=\{A_1,A_2,...,A_n\}$
Attribute Set of Interval Vague Values	$C=\{C_1,C_2,...,C_n\}$
Individual Interval Vague Value	$\phi_{ij}=\langle t_{ij}, f_{ij}\rangle, t_{ij}^{+}+f_{ij}^{+}\le 1$
Decision Alternative satisfying the Attribute	$\tilde{t}_{ij}=\left[t_{ij}^{-},t_{ij}^{+}\right]\subseteq[0,1]$
Decision Alternative not satisfying the Attribute	$f_{ij}=\left[f_{ij}^{-},f_{ij}^{+}\right]\subseteq[0,1]$
Attribute Weights	$W=(w_1,w_2,...,w_n)$, $w_j=\langle\tilde{t}_{wj}, f_{wj}\rangle, t_{wj}^{+}+f_{wj}^{+}\le 1$
Truth Membership of Attribute Weights	$\tilde{t}_{wj}=\left[t_{wj}^{-},t_{wj}^{+}\right]\subseteq[0,1]$
False Membership of Attribute Weights	$f_{wj}=\left[f_{wj}^{-},f_{wj}^{+}\right]\subseteq[0,1]$
Decision Matrix	$B=(\phi_{ij})_{m\times n}$
Weighted Decision Matrix	$B=(b_{ij})_{m\times n}$

Definition 1: (Zhou & Wu, 2006)

Suppose A and B are interval vague sets

$$A=\left\{\left\langle\left[t_A^-(x),t_A^+(x)\right],\left[f_A^-(x),f_A^+(x)\right]\right\rangle / x\in X\right\}, B=\left\{\left\langle\left[t_B^-(x),t_B^+(x)\right],\left[f_B^-(x),f_B^+(x)\right]\right\rangle / x\in X\right\}.$$

Then the distance between the interval vague sets A and B is defined as follows:

$$d(A,B)=\frac{1}{4n}\sum_{i=1}^{n}\left(\begin{array}{c}\left|t_A^-(x_i)-t_B^-(x_i)\right|+\left|t_A^+(x_i)-t_B^+(x_i)\right|+\left|f_A^-(x_i)-f_B^-(x_i)\right|+\left|f_A^+(x_i)-f_B^+(x_i)\right|\\+\left|\pi_A^-(x_i)-\pi_B^-(x_i)\right|+\left|\pi_A^+(x_i)-\pi_B^+(x_i)\right|\end{array}\right) \quad (5)$$

The weighted attribute value for the decision matrix $B=(\phi_{ij})_{m\times n}$ of each interval vague value ϕ_{ij} is given as follows:

$$b_{ij}=w_j\phi_{ij}=\left\langle\tilde{t}_{b_{ij}}, f_{b_{ij}}\right\rangle=\left\langle\left[t_{b_{ij}}^-,t_{b_{ij}}^+\right],\left[f_{b_{ij}}^-,f_{b_{ij}}^+\right]\right\rangle, \quad (6)$$

Where, $\tilde{t}_{bij}=\left[t_{ij}^-t_{wj}^-, t_{ij}^+t_{wj}^+\right],$ (7)

$$f_{bij}=\left[f_{ij}^-+f_{wj}^--f_{ij}^-f_{wj}^-, f_{ij}^++f_{wj}^+-f_{ij}^+f_{wj}^+\right], \quad (8)$$

The positive ideal solution is the best solution that is assumed (V^+). Each indicator value is the best value of the optional schemes.

The interval vague set positive ideal solution V^+ is given as:

$$\max\_i=\max_i(k_{ij})$$

$$V^+=\left\langle[t_{V_j^+}^-,t_{V_j^+}^+],[f_{V_j^+}^-,f_{V_j^+}^+]\right\rangle=\left\langle[t_{b_{ij}^{\max}}^-,t_{b_{ij}^{\max}}^+],[f_{b_{ij}^{\max}}^-,f_{b_{ij}^{\max}}^+]\right\rangle, \quad (9)$$

where $b_{ij}^{\max}$ refers to the b_{ij} corresponding to the maximum value obtained from the correlation coefficient k_{ij} between each b_{ij} and $\tilde{r}^+=([1,1],[0,0])$.

The negative ideal solution is another worst solution that is assumed (V^-). Each indicator value is the worst value of the optional projects.

The interval vague set negative ideal solution V^- is given as:

$$\min\_i=\min_i(k_{ij})$$

$$V^-=\left\langle[t_{V_j^-}^-,t_{V_j^-}^+],[f_{V_j^-}^-,f_{V_j^-}^+]\right\rangle=\left\langle[t_{b_{ij}^{\min}}^-,t_{b_{ij}^{\min}}^+],[f_{b_{ij}^{\min}}^-,f_{b_{ij}^{\min}}^+]\right\rangle, \quad (10)$$

where $b_{ij}^{\min}$ refers to the b_{ij} corresponding to the minimum value obtained from the correlation coefficient k_{ij} between each b_{ij} and $\tilde{r}^+=([1,1],[0,0])$. V^+ and V^- are compared with each interval vague value in the original project set. The correlation coefficient is used to confirm the order of the alternatives.

Model-1: The TOPSIS Algorithm with correlation coefficient of IVSs for bothIdealSolutions & Closeness Coefficient

Step-1: Calculate the weighted attribute value $b_{ij} = w_j \phi_{ij}$ of each interval vague value given in the decision matrix $B = \left(\phi_{ij}\right)_{m\times n}$.

Step-2: Calculate the Correlation coefficient k_{ij} between the individual interval vague values and the perfect positive vague value $\tilde{r}^{+} = \left([1,1],[0,0]\right)$, and form the corresponding correlation coefficient matrix $K = \left(k_{ij}\right)_{m\times n}$:

$$k_{ij} = k_{IVS}(b_{ij}, \tilde{r}^{+}) = \frac{C_{IVS}(b_{ij}, \tilde{r}^{+})}{\sqrt{E_{IVS}(b_{ij}).E_{IVS}(\tilde{r}^{+})}}$$

Step-3: Confirm the positive ideal solution V^{+} and the negative ideal solution V^{-} of the evaluation object based on the calculated Correlation coefficient k_{ij}.

Step-4: Calculate the correlation coefficient between each value b_{ij} and the positive ideal solution, as follows:

$$k_i^{+}(b_{ij}, V^{+}) = \frac{C_{IVS}(b_{ij}, V^{+})}{\sqrt{E_{IVS}(b_{ij}).E_{IVS}(V^{+})}}$$

Step-5: Calculate the correlation coefficient between each value b_{ij} and the negative ideal solution, as follows:

$$k_i^{-}(b_{ij}, V^{-}) = \frac{C_{IVS}(b_{ij}, V^{-})}{\sqrt{E_{IVS}(b_{ij}).E_{IVS}(V^{-})}}$$

Step-6: Confirm the relative adjacent degree and rank the alternatives based on the highest degree.The relative adjacent degree of the evaluation object and the ideal solution is:

$$D_i = \frac{K_i^{-}}{K_i^{+} + K_i^{-}} \quad i = 1, 2, ..., m \qquad (11)$$

Where $K_i^{-} = 1 - k_i^{-}$ and $K_i^{+} = 1 - k_i^{+}$.

(With regard to the relative adjacency relationship in analyzing how linearly the objects are interrelated, requires the computational property $K_i^{-} = 1 - k_i^{-}$ and $K_i^{+} = 1 - k_i^{+}$, with respect to the maximum value 1)

Model-2: The TOPSIS Algorithm with correlation coefficient of IVSs for bothIdeal Solutions andDistance Function for Closeness Coefficient

Step-1: Calculate the weighted attribute value $b_{ij} = w_j \phi_{ij}$ of each interval vague value given in the decision matrix $B = \left(\phi_{ij}\right)_{m\times n}$.

Step-2: Calculate the Correlation coefficient k_{ij} between the individual interval vague values and the perfect positive vague value $\tilde{r}^{+} = \left([1,1],[0,0]\right)$, and form the corresponding correlation coefficient matrix $K = \left(k_{ij}\right)_{m\times n}$, where

$$k_{ij} = k_{IVS}(b_{ij}, \tilde{r}^{+}) = \frac{C_{IVS}(b_{ij}, \tilde{r}^{+})}{\sqrt{E_{IVS}(b_{ij}).E_{IVS}(\tilde{r}^{+})}}$$

Step-3: Confirm the positive ideal solution V^{+} and the negative ideal solution V^{-} of the evaluation object based on the calculated correlation coefficient k_{ij}.

Step-4: Calculate the distance between each value b_{ij} and the positive ideal solution (equation 5).

Step-5: Calculate the distance between each value b_{ij} and the negative ideal solution (equation 5).

Step-6: Confirm the relative adjacent degree and rank alternatives based on the highest degree.
The relative adjacent degree of the evaluation object and the ideal solution is:

$$A_i = \frac{d_i^{-}}{d_i^{+} + d_i^{-}} \quad i = 1, 2, ..., m \qquad (12)$$

Numerical Illustration

A college intends to select a person for the position of Assistant Professor. Four aspects of the candidate are evaluated by experts,which are as follows:
C_1-Moral quality, C_2-Professional ability, C_3 - Creative ability,

$$\bar{B} = \begin{pmatrix} ([0.65,0.72],[0.22,0.27]) & ([0.52,0.63],[0.16,0.32]) & ([0.62,0.71],[0.23,0.28]) & ([0.32,0.43],[0.21,0.29]) \\ ([0.46,0.52],[0.34,0.41]) & ([0.73,0.81],[0.12,0.18]) & ([0.56,0.61],[0.25,0.29]) & ([0.41,0.51],[0.24,0.38]) \\ ([0.52,0.60],[0.33,0.40]) & ([0.33,0.45],[0.26,0.41]) & ([0.62,0.76],[0.13,0.21]) & ([0.53,0.62],[0.27,0.31]) \\ ([0.44,0.53],[0.30,0.45]) & ([0.38,0.50],[0.27,0.35]) & ([0.43,0.64],[0.24,0.32]) & ([0.61,0.72],[0.17,0.21]) \\ ([0.51,0.58],[0.35,0.40]) & ([0.64,0.80],[0.13,0.19]) & ([0.38,0.58],[0.22,0.38]) & ([0.58,0.65],[0.23,0.31]) \end{pmatrix}$$

$$B = \begin{pmatrix} ([0.195,0.288],[0.610,0.708]) & ([0.312,0.441],[0.244,0.456]) & ([0.372,0.497],[0.384,0.496]) & ([0.096,0.172],[0.605,0.716]) \\ ([0.138,0.208],[0.670,0.764]) & ([0.438,0.567],[0.208,0.344]) & ([0.336,0.427],[0.400,0.503]) & ([0.123,0.204],[0.620,0.752]) \\ ([0.156,0.240],[0.665,0.760]) & ([0.198,0.315],[0.334,0.528]) & ([0.372,0.532],[0.304,0.447]) & ([0.159,0.248],[0.635,0.724]) \\ ([0.132,0.212],[0.650,0.780]) & ([0.228,0.350],[0.343,0.480]) & ([0.258,0.448],[0.392,0.524]) & ([0.183,0.288],[0.585,0.684]) \\ ([0.153,0.232],[0.675,0.760]) & ([0.384,0.560],[0.217,0.352]) & ([0.228,0.406],[0.376,0.566]) & ([0.174,0.260],[0.615,0.724]) \end{pmatrix}$$

C_4- Knowledge range.

The experts provide evaluation data and weights to each aspect and they are all denoted by an interval vague value, namely, the interval number of the support degree given, and the interval number of the object degree, also given.The evaluation

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data and attribute weight are shown as follows. The order of the 5 candidates must be confirmed.

The evaluation data of different candidates given by experts are as follows:

$$K_{ij}=\begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14}\\ k_{21} & k_{22} & k_{23} & k_{24}\\ k_{31} & k_{32} & k_{33} & k_{34}\\ k_{41} & k_{42} & k_{43} & k_{44}\\ k_{51} & k_{52} & k_{53} & k_{54}\end{pmatrix}=\begin{pmatrix} \boxed{0.3370} & 0.6079 & 0.6717 & \underline{0.1877}\\ 0.2299 & \boxed{0.7905} & 0.6098 & 0.2230\\ 0.2629 & \underline{0.4127} & \boxed{0.7063} & 0.2799\\ \underline{0.2277} & 0.4766 & 0.5517 & \boxed{0.3362}\\ 0.2549 & 0.7468 & \underline{0.4902} & 0.3002\end{pmatrix}$$

The attribute weight given by the experts as follows:

$$W=\{([0.3,0.4],[0.5,0.6]),\ ([0.6,0.7],[0.1,0.2]),\ ([0.6,0.7],[0.2,0.3]),\ ([0.3,0.4],[0.5,0.6])\}$$

Algorithm usingModel-1

Step-1:Calculate the weighted $\tilde{b}_{ij}$ as in equations (5.2) to (5.4) from the decision matrix B

$$\tilde{b}_{ij}=w_j\phi_{ij}$$

Where, $\tilde{t}_{b_{ij}}=\left\langle\left[t_{ij}^{-}t_{wj}^{-},t_{ij}^{+}t_{wj}^{+}\right]\right\rangle$, $f_{b_{ij}}=\left\langle\left[f_{ij}^{-}+f_{wj}^{-}-f_{ij}^{-}f_{wj}^{-},\ f_{ij}^{+}+f_{wj}^{+}-f_{ij}^{+}+f_{wj}^{+}\right]\right\rangle$,

$$\tilde{t}_{b_{11}}=\left(\left[t_{11}^{-}t_{w1}^{-},t_{11}^{+}t_{w1}^{+}\right]\right)=\left(\left[(0.65)(0.3),(0.72)(0.4)\right]\right)=\left([0.195,0.288]\right)$$

Similarly the other values can be calculated and are given as follows:

Step-2: Calculate the Correlation coefficient k_{ij} between the individual interval vague values b_{ij} of the matrix $B$and the perfect positive vague value $\tilde{r}^{+}=([1,1],[0,0])$:

Consider the interval vague value $b_{11}=([0.195,0.288],[0.610,0.704])$.

$$E_{IVS}(b_{11},b_{11})=0.5134,\ E_{IVS}(\tilde{r}^{+},\tilde{r}^{+})=1,\ C_{IVS}(b_{11},\tilde{r}^{+})=0.2415,\ k_{IVS}(b_{11},\tilde{r}^{+})=0.3370.$$

Hence $k_{11}=0.3370$.

Similarly the correlation coefficient for all the other entries can be calculated.

(The positive ideal solution is boxed and the negative ideal solution underlined)

Step-3: Confirm the ideal solution and the negative solution of the evaluation object.The vague set positive ideal solution V^{+} and the negative ideal solution V^{-} are shown as follows:

$$V^{+}=\{([0.195,0.288],[0.610,0.704]),([0.438,0.567],[0.208,0.344]),([0.372,0.532],[0.304,0.447]),([0.183,0.288],[0.585,0.684])\}$$

$$V^{-}=\{([0.132,0.212],[0.650,0.780]),([0.198,0.315],[0.334,0.528]),([0.228,0.406],[0.376,0.566]),([0.096,0.172],[0.605,0.716])\}$$

Step-4: Calculate the correlation coefficient between each interval vague value of the matrix B and the positive ideal solution,

$$E_{IVS}(b_{ij})=\frac{1}{2}\sum_{j=1}^{n}\left((t_{b_{ij}}^{-})^2+(t_{b_{ij}}^{+})^2+(1-f_{b_{ij}}^{-})^2+(1-f_{b_{ij}}^{+})^2+(\pi_{b_{ij}}^{-})^2+(\pi_{b_{ij}}^{+})^2\right)$$

$$E_{IVS}(V^{+})=\frac{1}{2}\sum_{j=1}^{n}\left((t_{V_j^{+}}^{-})^2+(t_{V_j^{+}}^{+})^2+(1-f_{V_j^{+}}^{-})^2+(1-f_{V_j^{+}}^{+})^2+(\pi_{V_j^{+}}^{-})^2+(\pi_{V_j^{+}}^{+})^2\right)$$

$$C_{IVS}(b_{ij},V^{+})=\frac{1}{2}\sum_{j=1}^{n}\left((t_{b_{ij}}^{-}t_{V_j^{+}}^{-})+(t_{b_{ij}}^{+}t_{V_j^{+}}^{+})+(1-f_{b_{ij}}^{-})(1-f_{V_j^{+}}^{-})+(1-f_{b_{ij}}^{+})(1-f_{V_j^{+}}^{+})+(\pi_{b_{ij}}^{-}\pi_{V_j^{+}}^{-})+(\pi_{b_{ij}}^{+}\pi_{V_j^{+}}^{+})\right)$$

$$k_i^{+}=k_{IVS}(b_{ij},V^{+})=\frac{C_{IVS}(b_{ij},V^{+})}{\sqrt{E_{IVS}(b_{ij}).E_{IVS}(V^{+})}}$$

Where, $\pi_{b_{ij}}^{-}(x)=1-t_{b_{ij}}^{+}(x)-f_{b_{ij}}^{+}(x)$, $\pi_{b_{ij}}^{+}(x)=1-t_{b_{ij}}^{-}(x)-f_{b_{ij}}^{-}(x)$.

The entries of b_{1j} in the matrix $B$and the positive and negative ideal solutions taken in the order $t^{-},t^{+},f^{-},f^{+},\pi^{-},\pi^{+}$ are given as follows:

The entries of b_{1j} in the matrix B

0.195	0.288	0.610	0.704	0.004	0.195
0.312	0.441	0.244	0.456	0.103	0.444
0.372	0.497	0.384	0.496	0.007	0.244
0.096	0.172	0.605	0.716	0.112	0.299

The entries of positive ideal solution V^{+}

0.195	0.288	0.610	0.704	0.004	0.195
0.438	0.567	0.208	0.344	0.089	0.354
0.372	0.532	0.304	0.447	0.021	0.324
0.183	0.288	0.585	0.684	0.028	0.232

Calculating the correlation coefficient between the entries of b_{1j} and the positive ideal solution, the values can be obtained as follows:

$$E_{IVS}(b_{1j})=1.8257,\ E_{IVS}(V^{+})=1.8174,\ C_{IVS}(b_{1j},V^{+})=1.7984$$

$$k_1^{+}=k_{IVS}(b_{1j},V^{+})=\frac{C_{IVS}(b_{1j},V^{+})}{\sqrt{E_{IVS}(b_{1j}).E_{IVS}(V^{+})}}=0.9873$$

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Similarly all the correlation coefficients can be calculated, and given as follows:

$k_2^+ = k_{IVS}(b_{2j}, V^+) = 0.9887,$

$k_3^+ = k_{IVS}(b_{3j}, V^+) = 0.9725,$

$k_4^+ = k_{IVS}(b_{4j}, V^+) = 0.9737,$

$k_5^+ = k_{IVS}(b_{5j}, V^+) = 0.9892.$

$K_1^+ = 0.0127,\ K_2^+ = 0.0113,\ K_3^+ = 0.0275,$

$K_4^+ = 0.0263,\ K_5^+ = 0.0108.$

where $K_i^+ = 1 - k_i^+$.

Step-5:Calculate the correlation coefficient between each interval vague value of the matrix *B* and the negative ideal solution,

$$E_{IVS}(b_{ij}) = \frac{1}{2}\sum_{j=1}^{n}\left((t_{b_{ij}}^-)^2 + (t_{b_{ij}}^+)^2 + (1 - f_{b_{ij}}^-)^2 + (1 - f_{b_{ij}}^+)^2 + (\pi_{b_{ij}}^-)^2 + (\pi_{b_{ij}}^+)^2\right)$$

$$E_{IVS}(V^-) = \frac{1}{2}\sum_{j=1}^{n}\left((t_{V_j^-}^-)^2 + (t_{V_j^-}^+)^2 + (1 - f_{V_j^-}^-)^2 + (1 - f_{V_j^-}^+)^2 + (\pi_{V_j^-}^-)^2 + (\pi_{V_j^-}^+)^2\right)$$

$$C_{IVS}(b_{ij}, V^-) = \frac{1}{2}\sum_{j=1}^{n}\left((t_{b_{ij}}^- t_{V_j^-}^-) + (t_{b_{ij}}^+ t_{V_j^-}^+) + (1 - f_{b_{ij}}^-)(1 - f_{V_j^-}^-) + (1 - f_{b_{ij}}^+)(1 - f_{V_j^-}^+) + (\pi_{b_{ij}}^- \pi_{V_j^-}^-) + (\pi_{b_{ij}}^+ \pi_{V_j^-}^+)\right)$$

$$k_i^- = k_{IVS}(b_{ij}, V^-) = \frac{C_{IVS}(b_{ij}, V^-)}{\sqrt{E_{IVS}(b_{ij}).E_{IVS}(V^-)}}.$$

The entries of b_{1j} in the matrix *B*

0.195	0.288	0.610	0.704	0.004	0.195
0.312	0.441	0.244	0.456	0.103	0.444
0.372	0.497	0.384	0.496	0.007	0.244
0.096	0.172	0.605	0.716	0.112	0.299

The entries of negative ideal solution V^-

0.132	0.212	0.650	0.780	0.008	0.218
0.198	0.315	0.334	0.528	0.157	0.468
0.228	0.406	0.376	0.566	0.028	0.396
0.096	0.172	0.605	0.716	0.112	0.299

Calculating the correlation coefficient between the entries of b_{1j} and the negative ideal solution, the values are obtained as follows:

$E_{IVS}(b_{1j}) = 1.8257,\ \ E_{IVS}(V^-) = 1.8843,\ \ C_{IVS}(b_{1j}, V^-) = 1.8248$

$$k_1^- = k_{IVS}(b_{1j}, V^-) = \frac{C_{IVS}(b_{1j}, V^-)}{\sqrt{E_{IVS}(b_{1j}).E_{IVS}(V^-)}} = 0.9838$$

Similarly all the correlation coefficients can be calculated, and given as follows:

$k_2^- = k_{IVS}(b_{2j}, V^-) = 0.9689,$

$k_3^- = k_{IVS}(b_{3j}, V^-) = 0.9879,$

$k_4^- = k_{IVS}(b_{4j}, V^-) = 0.9937,$

$k_5^- = k_{IVS}(b_{5j}, V^-) = 0.9725.$

$K_1^- = 0.0162,\ K_2^- = 0.0311,\ K_3^- = 0.0121,$

$K_4^- = 0.0063,\ K_5^- = 0.0275.$

Where $K_i^- = 1 - k_i^-$.

Step-6: Confirm the relative adjacent degree and rank alternatives based on the highest degree. The relative adjacent degree of the evaluation object and the ideal solution are:

$$A_i = \frac{K_i^-}{K_i^+ + K_i^-} \quad i = 1, 2, ..., m$$

$$A_1 = \frac{K_1^-}{K_1^+ + K_1^-} = 0.5605,$$

$$A_2 = \frac{K_2^-}{K_2^+ + K_2^-} = 0.7335,$$

$$A_3 = \frac{K_3^-}{K_3^+ + K_3^-} = 0.3055,$$

$$A_4 = \frac{K_4^-}{K_4^+ + K_4^-} = 0.1932,$$

$$A_5 = \frac{K_5^-}{K_5^+ + K_5^-} = 0.7180.$$

Ranking alternatives based on the relative adjacent degree, it follows that:

$$A_2 > A_5 > A_1 > A_3 > A_4.$$

Hence A_2 is the best alternative.

Algorithm using Model-2:

Step-1 to Step-3 of the numerical illustration for Model-2 is same as that of the numerical illustration for Model-1, which is clear from the algorithm given for both models.

Step-4:Calculate the distance between each interval vague value of the matrix *B* and the positive ideal solution,

$$d_i^+ = \frac{1}{4n}\sum_{j=1}^{n}\left(\left|t_{b_{ij}}^- - t_{V_j^+}^-\right| + \left|t_{b_{ij}}^+ - t_{V_j^+}^+\right| + \left|f_{b_{ij}}^- - f_{V_j^+}^-\right| + \left|f_{b_{ij}}^+ - f_{V_j^+}^+\right| + \left|\pi_{b_{ij}}^- - \pi_{V_j^+}^-\right| + \left|\pi_{b_{ij}}^+ - \pi_{V_j^+}^+\right|\right)$$

$$d_1^+ = d_1^+(b_{1j}, V^+) = 0.0730,$$
$$d_2^+ = d_2^+(b_{2j}, V^+) = 0.0606,$$
$$d_3^+ = d_3^+(b_{3j}, V^+) = 0.1244,$$
$$d_4^+ = d_4^+(b_{4j}, V^+) = 0.0955,$$
$$d_5^+ = d_5^+(b_{5j}, V^+) = 0.0704.$$

Step-5:Calculate the distance between each interval vague value of the matrix *B* and the negative ideal solution,

$$d_i^- = \frac{1}{4n}\sum_{j=1}^{n}\left(\left|t_{b_{ij}}^- - t_{V_j^-}^-\right| + \left|t_{b_{ij}}^+ - t_{V_j^-}^+\right| + \left|f_{b_{ij}}^- - f_{V_j^-}^-\right| + \left|f_{b_{ij}}^+ - f_{V_j^-}^+\right| + \left|\pi_{b_{ij}}^- - \pi_{V_j^-}^-\right| + \left|\pi_{b_{ij}}^+ - \pi_{V_j^-}^+\right|\right)$$

$$d_1^- = d_1^-(b_{1j}, V^-) = 0.0784,$$
$$d_2^- = d_2^-(b_{2j}, V^-) = 0.1054,$$
$$d_3^- = d_3^-(b_{3j}, V^-) = 0.0643,$$
$$d_4^- = d_4^-(b_{4j}, V^-) = 0.0473,$$
$$d_5^- = d_5^-(b_{5j}, V^-) = 0.0901.$$

Step-6: Confirm the relative adjacent degree and rank alternatives based on the highest degree. The relative adjacent degree of the evaluation object and the ideal solution are:

$$A_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad i = 1, 2, ..., m$$

$$A_1 = \frac{d_1^-}{d_1^+ + d_1^-} = 0.5178,$$

$$A_2 = \frac{d_2^-}{d_2^+ + d_2^-} = 0.6349,$$

$$A_3 = \frac{d_3^-}{d_3^+ + d_3^-} = 0.3407,$$

$$A_4 = \frac{d_4^-}{d_4^+ + d_4^-} = 0.3312,$$

$$A_5 = \frac{d_5^-}{d_5^+ + d_5^-} = 0.5614.$$

Ranking the alternatives based on the relative adjacent degree, it follows that:

$$A_2 > A_5 > A_1 > A_3 > A_4.$$

Hence A_2 is the best alternative.

Comparison of Proposed Topsis with Existing Ranking Methods. The proposed TOPSIS algorithm is compared with the previous methods of score and accuracy functions and presented as follows:

Definition 2: (Chen & Tan, 1994)

Let $A = \left\langle \left[t_{ij}^-, t_{ij}^+\right], \left[f_{ij}^-, f_{ij}^+\right] \right\rangle$ be an interval vague value. Then the score function for the interval vague value *A* is defined as: $S_{ij} = \dfrac{t_{ij}^- + t_{ij}^+}{2} - \dfrac{f_{ij}^- + f_{ij}^+}{2}$ (13)

Definition 3: (Hong & Choi, 2000)

Let $A = \left\langle \left[t_{ij}^-, t_{ij}^+\right], \left[f_{ij}^-, f_{ij}^+\right] \right\rangle$ be an interval vague value. Then the score function for the interval vague value *A* is defined as: $H_{ij} = \dfrac{t_{ij}^- + t_{ij}^+}{2} + \dfrac{f_{ij}^- + f_{ij}^+}{2}$ (14)

Xu, (2007e) also defined a same kind of function for IVIFSs and named it accuracy function which is given as follows:

Definition 4: (Xu, 2007)

Let $A = \left\langle [a,b],[c,d] \right\rangle$ be an interval valued intuitionistic fuzzy number. Then the accuracy function for the interval valued intuitionistic fuzzy number *A* is defined as follows:

$$H(A) = \frac{a+b+c+d}{2} \quad (15)$$

Definition 5: (Liu, 2009)

Let $A = \left\langle \left[t_{ij}^-, t_{ij}^+\right], \left[f_{ij}^-, f_{ij}^+\right] \right\rangle$ be an interval vague value. Then the score function for the interval vague value *A* is defined as follows:

$$L_{ij} = \left(t_{ij}^* + t_{ij}^*.\pi_{ij}^*\right) - \left(f_{ij}^* + f_{ij}^*.\pi_{ij}^*\right) = \left(t_{ij}^* - f_{ij}^*\right)\left(1 + \pi_{ij}^*\right) \quad (16)$$

Where,

$$t_{ij}^* = \frac{t_{ij}^- + t_{ij}^+}{2}, \quad f_{ij}^* = \frac{f_{ij}^- + f_{ij}^+}{2}, \quad \pi_{ij}^* = \frac{\pi_{ij}^- + \pi_{ij}^+}{2}.$$

Nayagam et al, (2011) proved the invalidity of the Chen & Tan, (1994), Hong & Choi, (2000) and the Xu, (2007) score and accuracy functions and suggested a novel and reasonable accuracy function which claims the comparability of all interval valued intuitionistic fuzzy numbers. Their accuracy function is as follows:

Definition 6: (Nayagam et al, 2011)

Let $A = \left\langle [a,b],[c,d] \right\rangle$ be an interval valued intuitionistic fuzzy number. Then the accuracy function for the interval valued intuitionistic fuzzy number *A* is defined as follows:

$$L(A) = \frac{a+b-d(1-b)-c(1-a)}{2} \quad (17)$$

The distance function used in Zhou & Wu, (2006) is utilized for all the comparison methods to calculate the closeness coefficient.

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Comparison with the Score Function of Chen & Tan, (1994)
The TOPSIS Algorithm with the Score Function of Chen & Tan, (1994) to identify the ideal solutions is given as follows:
Calculate the Score function S_{ij} for each individual interval vague values.

$$S_{ij} = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \\ s_{51} & s_{52} & s_{53} & s_{54} \end{pmatrix} = \begin{pmatrix} \boxed{-0.4175} & 0.0265 & -0.0055 & \underline{-0.5265} \\ \underline{-0.5440} & \boxed{0.2265} & -0.0700 & -0.5225 \\ -0.5145 & \underline{-0.1745} & \boxed{0.0765} & -0.4760 \\ -0.5430 & -0.1225 & -0.1050 & \boxed{-0.3990} \\ -0.5250 & 0.1875 & \underline{-0.1540} & -0.4525 \end{pmatrix}$$

Confirm the ideal solution and the negative solution of the evaluation object using the above Score function value obtained from step-2. The vague set ideal solution V^+ and the negative ideal solution V^- is shown as follows:

$$V^+ = \{([0.195, 0.288],[0.610, 0.704]),([0.438, 0.567],[0.208, 0.344]), ([0.372, 0.532],[0.304, 0.447]),([0.183, 0.288],[0.585, 0.684])\}$$

$$V^- = \{([0.138, 0.208],[0.670, 0.764]),([0.198, 0.315],[0.334, 0.528]), ([0.228, 0.406],[0.376, 0.566]),([0.096, 0.172],[0.605, 0.716])\}$$

Calculate the distance between each value b_{ij} and the positive ideal solution, as follows:

$$d_1^+ = d_1^+(b_{1j}, V^+) = 0.0730,$$
$$d_2^+ = d_2^+(b_{2j}, V^+) = 0.0669,$$
$$d_3^+ = d_3^+(b_{3j}, V^+) = 0.0866,$$
$$d_4^+ = d_4^+(b_{4j}, V^+) = 0.0955,$$
$$d_5^+ = d_5^+(b_{5j}, V^+) = 0.0704,$$

Calculate the distance between each value b_{ij} and the negative ideal solution, as follows:

$$d_1^- = d_1^-(b_{1j}, V^-) = 0.0779,$$
$$d_2^- = d_2^-(b_{2j}, V^-) = 0.1006,$$
$$d_3^- = d_3^-(b_{3j}, V^-) = 0.0621,$$
$$d_4^- = d_4^-(b_{4j}, V^-) = 0.0530,$$
$$d_5^- = d_5^-(b_{5j}, V^-) = 0.0874,$$

Confirm the relative adjacent degree and rank alternatives based on the highest degree. The relative adjacent degree of the evaluation object and the ideal solution are:

$$A_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad i = 1, 2, ..., m$$
$$A_1 = 0.5162,\ A_2 = 0.6006,\ A_3 = 0.4176,$$
$$A_4 = 0.3569,\ A_5 = 0.5538.$$

Ranking alternatives based on the relative adjacent degree, it follows that:

$$A_2 > A_5 > A_1 > A_3 > A_4.$$

Hence A_2 is the best alternative.

$$H_{ij} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \\ h_{51} & h_{52} & h_{53} & h_{54} \end{pmatrix} = \begin{pmatrix} 0.9005 & 0.7265 & 0.8475 & \underline{0.7945} \\ 0.8900 & \boxed{0.7785} & 0.8215 & 0.8495 \\ \boxed{0.9105} & \underline{0.6875} & \boxed{0.8275} & 0.8830 \\ \underline{0.8870} & 0.7005 & 0.8110 & 0.8700 \\ 0.9100 & 0.7565 & \underline{0.7880} & \boxed{0.8865} \end{pmatrix}$$

Comparison with the Score Function of Hong & Choi, (2000)
The TOPSIS Algorithm with the Score Function of Hong & Choi, (2000)to identify the ideal solutions is given as follows:

Calculate the Score function S_{ij} for each individual Interval vague values.

Confirm the ideal solution and the negative solution of the evaluation object using the above Score function value obtained from step-2. The vague set ideal solution V^+ and the negative ideal solution V^- is shown as follows:

$$V^+ = \{([0.156, 0.240],[0.665, 0.760]),([0.438, 0.567],[0.208, 0.344]), ([0.372, 0.532],[0.304, 0.447]),([0.174, 0.260],[0.615, 0.724])\}$$

$$V^- = \{([0.132, 0.212],[0.650, 0.780]),([0.198, 0.315],[0.334, 0.528]), ([0.228, 0.406],[0.376, 0.566]),([0.096, 0.172],[0.605, 0.716])\}$$

Calculate the distance between each value $b_{ij}$and the positive ideal solution as follows:

$$d_1^+ = d_1^+(b_{1j}, V^+) = 0.0684,$$
$$d_2^+ = d_2^+(b_{2j}, V^+) = 0.0391,$$
$$d_3^+ = d_3^+(b_{3j}, V^+) = 0.0816,$$
$$d_4^+ = d_4^+(b_{4j}, V^+) = 0.0909,$$
$$d_5^+ = d_5^+(b_{5j}, V^+) = 0.0454,$$

Calculate the distance between each value b_{ij} and the negative ideal solution as follows:

$$d_1^- = d_1^-(b_{1j}, V^-) = 0.0777,$$
$$d_2^- = d_2^-(b_{2j}, V^-) = 0.1064,$$
$$d_3^- = d_3^-(b_{3j}, V^-) = 0.0642,$$
$$d_4^- = d_4^-(b_{4j}, V^-) = 0.0472,$$
$$d_5^- = d_5^-(b_{5j}, V^-) = 0.0901,$$

Confirm the relative adjacent degree and rank alternatives based on the highest degree. The relative adjacent degree of the evaluation object and the ideal solution is:

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$$A_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad i = 1, 2, ..., m$$

$$A_1 = 0.5318,\ A_2 = 0.7313,\ A_3 = 0.4403,$$
$$A_4 = 0.3418,\ A_5 = 0.6649.$$

Ranking the alternatives based on the relative adjacent degree, it follows that:

$$A_2 > A_5 > A_1 > A_3 > A_4.$$

Hence A_2 is the best alternative.

Comparison with the Score Function of Liu, (2009)

The TOPSIS Algorithm with the Score Function of Liu, (2009) to identify the ideal solutions is given as follows:Calculate the Score function S_{ij} for each individual Interval vague values.

$$L_{ij} = \begin{pmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ l_{21} & l_{22} & l_{23} & l_{24} \\ l_{31} & l_{32} & l_{33} & l_{34} \\ l_{41} & l_{42} & l_{43} & l_{44} \\ l_{51} & l_{52} & l_{53} & l_{54} \end{pmatrix} = \begin{pmatrix} \boxed{-0.4599} & 0.0337 & -0.0062 & \underline{-0.6347} \\ -0.6038 & \boxed{0.2766} & -0.0817 & -0.6273 \\ -0.5605 & \underline{-0.2290} & \boxed{0.0899} & -0.5317 \\ -0.6044 & -0.1592 & \underline{-0.1248} & \boxed{-0.4508} \\ \underline{-0.6195} & 0.2256 & 0.1866 & -0.5038 \end{pmatrix}$$

Confirm the positive and the negative ideal solutions of the evaluation object using the above Score function value obtained from step-2. The vague set ideal solution V^+ and the negative ideal solution V^- is shown as follows:

$$V^+ = \{([0.195, 0.288], [0.610, 0.708]), ([0.438, 0.567], [0.208, 0.344]), ([0.372, 0.532], [0.304, 0.447]), ([0.183, 0.288], [0.585, 0.684])\}$$

$$V^- = \{([0.153, 0.232], [0.675, 0.760]), ([0.198, 0.315], [0.334, 0.528]), ([0.258, 0.448], [0.392, 0.524]), ([0.096, 0.172], [0.605, 0.716])\}$$

Calculate the distance between each value b_{ij} and the positive ideal solutionas follows:

$$d_1^+ = d_1^+(b_{1j}, V^+) = 0.0730,$$
$$d_2^+ = d_2^+(b_{2j}, V^+) = 0.0669,$$
$$d_3^+ = d_3^+(b_{3j}, V^+) = 0.0866,$$
$$d_4^+ = d_4^+(b_{4j}, V^+) = 0.0955,$$
$$d_5^+ = d_5^+(b_{5j}, V^+) = 0.0704,$$

Calculate the distance between each value b_{ij} and the negative ideal solution as follows:

$$d_1^- = d_1^-(b_{1j}, V^-) = 0.0635,$$
$$d_2^- = d_2^-(b_{2j}, V^-) = 0.0977,$$
$$d_3^- = d_3^-(b_{3j}, V^-) = 0.0491,$$
$$d_4^- = d_4^-(b_{4j}, V^-) = 0.0445,$$
$$d_5^- = d_5^-(b_{5j}, V^-) = 0.0929,$$

Confirm the relative adjacent degree and rank alternatives based on the highest degree. The relative adjacent degree of the evaluation object and the ideal solution is:

$$A_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad i = 1, 2, ..., m$$

$$A_1 = 0.4652,\ A_2 = 0.5935,\ A_3 = 0.3618,$$
$$A_4 = 0.3178,\ A_5 = 0.5689.$$

Ranking alternatives based on the relative adjacent degree, it follows that:

$$A_2 > A_5 > A_1 > A_3 > A_4.$$

Hence A_2 is the best alternative.

Comparison with the Accuracy Function of Nayagam et al., (2011)

Proceeding with the same TOPSIS algorithm and using the Accuracy function of Nayagam et al., (2011) to identify the positive and negative ideal solutions, the same numerical results as in Chen & Tan, (1994) numerical illustration are obtained.The Score function $L(A)$ for each individual Interval vague values is given as follows:

$$L(A) = \begin{pmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \\ L_{51} & L_{52} & L_{53} & L_{54} \end{pmatrix} = \begin{pmatrix} \boxed{-0.2561} & 0.0751 & 0.1892 & \underline{-0.4359} \\ \underline{-0.4183} & \boxed{0.3696} & 0.1046 & -0.4077 \\ -0.3714 & \underline{-0.0583} & \boxed{0.2519} & -0.3357 \\ -0.4174 & -0.0006 & 0.0629 & \boxed{-0.2469} \\ -0.3852 & 0.3277 & \underline{0.0037} & -0.3049 \end{pmatrix}$$

The ranking of the alternatives is given as follows:

$$A_2 > A_5 > A_1 > A_3 > A_4.$$

Where, the best alternative is A_2.

Table-2: Comparison Table

TOPSIS METHODS	RANKING OF ALTERNATIVES
Proposed MODEL-1 (TOPSIS with correlation coefficient of IVSs for Ideal solutions & Closeness coefficient)	$A_2 > A_5 > A_1 > A_3 > A_4.$ The best alternative is A_2.
Proposed MODEL-2 (TOPSIS with correlation coefficient of IVSs for Ideal solutions & Distance function for Closeness coefficient)	$A_2 > A_5 > A_1 > A_3 > A_4.$ The best alternative is A_2.
Chen & Tan, (1994) Method of Score Function for Ideal Solutions	$A_2 > A_5 > A_1 > A_3 > A_4.$ The best alternative is A_2.
Hong & Choi, (2000) Method of Score Function for Ideal Solutions	$A_2 > A_5 > A_1 > A_3 > A_4.$ The best alternative is A_2.
Liu, P.D., (2009a) Method of Score Function for Ideal Solutions	$A_2 > A_5 > A_1 > A_3 > A_4.$ The best alternative is A_2.

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Nayagam et al., (2011) Method of Accuracy Function for Ideal Solutions	$A_2 > A_5 > A_1 > A_3 > A_4$. The best alternative is A_2.

From the numerical illustrations and comparisons made above, it can be observed that the final decision on the ranking of alternatives remains the same in all the TOPSIS methods. The proposed method differs from existing methods in identifying positive and negative ideal solutions, as presented clearly in *Table*-2 presents the details of the final order of ranking of alternatives. It is seen from the proposed model that correlation coefficient can also be used as a tool for identifying the positive and negative ideal solutions in TOPSIS methods. The positive and negative ideal solutions identified by using correlation coefficient differ from the positive and negative ideal solutions identified by using existing score and accuracy functions. For the positive ideal solution, computed through correlation coefficient, it is seen that its entries contain all the other entries of that particular attribute for all the five alternatives. For the negative ideal solution, computed through correlation coefficient, it is observed that its entries are contained in all the other entries of that particular attribute for all the five alternatives. This is an indication for a better ideal solution for any decision making system. Hence the proposed method of TOPSIS with correlation coefficient for identifying the ideal solutions is a better tool when compared with existing methods in literature.

II. CONCLUSION

This paper explored the multi-attribute decision making problem based on interval vague sets for TOPSIS. First, based on the operation rules of the interval vague sets, weighted operations to the interval vague attribute value are introduced. Then the positive and negative ideal solutions are confirmed on the basis of the correlation coefficient of IVSs instead of score functions used in literature. The relative adjacent degree is calculated in the TOPSIS algorithm using the same correlation coefficient of IVSs, and according to the calculated relative adjacent degree, the order of the alternatives is confirmed. Two different TOPSIS algorithms are proposed, Model-1 is the TOPSIS algorithm with correlation coefficient of IVSs for both ideal solutions and closeness coefficient and Model-2 is the TOPSIS algorithm with correlation coefficient of IVSs for ideal solutions and distance function for closeness coefficient.The numerical illustration proves the practicality of the proposed TOPSIS model. A detailed comparison is made with the existing methods of score and accuracy functions to identify positive and negative ideal solutions. The comparison study reveals the advantage of using correlation coefficient over the score and accuracy functions in identifying ideal solutions. The final ranking of the alternatives remains the same throughout all the methods as clearly presented in *Table*-2.

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