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Pair Sum Labeling Of Some Special Graphs

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Abstract - Let G be a (p, q) graph. A one-one map $f : V(G) \to \{\pm1, \pm2, \ldots, \pm p\}$ is said to be a pair sum labeling if the induced edge function, $f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{q/2}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{(q-1)/2}\} \cup \{k_{(q+1)/2}\}$ according as q is even or odd. Recently, the pair sum labeling is introduced by R.Ponraj, J. V. X. Parthipan. In this paper we study about the pair sum labeling of coconut tree CT(m, n), the the Y-tree Y_{n+1}, Jelly fish J(m, n), (m, 2)-kite, (m, 1)-kite and the theta graph $\Theta(l^{[m]})$, for m even.

Keywords: pair sum labeling, pair sum graph, AMS Classification: 05C78.

I. INTRODUCTION

The graph considered here are all finite, undirected and simple. V(G) and E(G) denote the vertex set and edge set of a graph G. The pair sum labeling is introduced in [3] by R. Ponraj and et al. In [3],[4] [5] and [6] they study the pair sum labeling of cycle, path, star and some of their related graphs like $B_{m,n}$, $K_{1,n} \cup K_{1,m}$, $P_m \cup K_{1,n}$, $C_n \cup C_n$ etc. In this paper we study pair sum labeling of coconut tree CT(m, n), the the Y-tree Y_{n+1}, Jelly fish J(m, n) , (m, 2)-kite, (m, 1)-kite and the theta graph $\Theta(l^{[m]})$, for m even. Let x be any real number, then $[x]$ denotes the largest integer less than or equal to x. Terms and terminology as in Harary [2].

Definition 1.1 [3]. Let G(V, E) be a (p, q) graph. A one-one function $f : V(G) \to \{\pm1, \pm2, \ldots, \pm p\}$ is said to be a pair sum labeling if the induced edge function $f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{q/2}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{(q-1)/2}\} \cup \{k_{(q+1)/2}\}$ according as q is even or odd. A graph with a pair sum labeling defined on it is called a pair sum graph.

Definition 1.2: A coconut Tree CT(m,n) is the graph obtained from the path P_m by appending n new pendent edges at an end vertex of P_m .

Definition 1.3: A Y-tree Y_{n+1} is a graph obtained from the path P_n by appending an edge to a vertex of the path $P_m$adjacent to an end point.

Definition 1.4: An (n, t)-kite is a cycle of length n with a t-edge path (the tail) attached to one vertex. In particular, the (n, 1)-kite is a cycle of length n with an edge attached to one vertex. (n, 1)-kite is also known as flag, $F\ell_n$.

Definition 1.5:The Jelly fish graph J(m, n) is obtained from a 4- cycle v_1, v_2, v_3, v_4 by joining v_1 and v_3 with an edge and appending m pendent edges to v_2 and n pendent edges to v_4.

Definition 1.6: Take k paths of length $l_1, l_2, l_3, \ldots, l_k$, where $k \geq 3$ and $l_i = 1$ forat most one i. Identify their end points to form a new graph. The new graph is called a generalized theta graph, and is denoted by $\Theta(l_1, l_2, l_3, \ldots, l_k)$. In other words, $\Theta(l_1, l_2, l_3, \ldots, l_k)$ consists $k \geq 3$ pair wise internally disjoint paths of length $l_1, l_2, l_3, \ldots, l_k$that share a pair of common end points u and v. If each *l_i (i = 1,2, . . . ,k) is equal to l, we will write* $\Theta(l^{[k]})$.

In [3], R.Ponraj and J. V. X. Parthipan have proved the following results which we will use in the proof of our theorem.

Theorem 1.1 (as Theorem 3.9 in [3]): (3, 1) - kite is not a pair sum graph.

Theorem 1.2[2]: The complete bipartite graphs $K_{1,n}$ and $K_{2,n}$ are pair sum graphs.

II. MAIN RESULTS

Theorem 2.1: The (m, 2)-kite is a pair sum graph for all $m \geq 3$.

Proof: Let V be the vertex set and E be the edge set of (m, 2)-kite. Then $|V| = |E| = m + 2$. We consider the following two cases.

Case1: m is even. Let m = 2r.

Let $V = \{u_i, v_i / 1 \leq i \leq r\} \cup \{w_1, w_2\}$ and $E = \{u_iu_{i+1}, v_iv_{i+1} / 1 \leq i \leq r-1\} \cup \{u_rv_1, v_ru_1, u_r w_1, w_1w_2\}$.

Define a map $f : V \to \{\pm1, \pm2, \ldots, \pm(2r+2)\}$ as follows:

$f(u_i) = 2i$; $1 \leq i \leq r-1$

$f(v_i) = -f(u_i)$; $1 \leq i \leq r-1$

$f(u_r) = 1$, $f(v_r) = -1$, $f(w_1) = -(r+1)$, $f(w_2) = (2r+1)$.

Then f is a pair sum labeling.

Case2: m is odd.

Subcase 2.1: $m \geq 9$

Let m = 2r+1 , $r \geq 4$.

Let $V = \{u_i / 1 \leq i \leq r\} \cup \{v_i / 1 \leq i \leq r+1\} \cup \{w_1, w_2\}$ and $E = \{u_iu_{i+1} / 1 \leq i \leq r-1\} \cup \{v_iv_{i+1} / 1 \leq i \leq r\} \cup \{u_rv_1, v_{r+1}u_1, v_r w_1, w_1w_2\}$.

Define $f : V \to \{\pm1, \pm2, \ldots, \pm(2r+3)\}$ as follows:

$f(u_i) = 2i$; $1 \leq i \leq r-1$

$f(v_i) = -f(u_i)$; $1 \leq i \leq r-2$

$f(u_r) = 2r-1$, $f(v_{r-1}) = 2r-3$, $f(v_r) = -(2r-2)$, $f(v_{r+1}) = -(2r-1)$

$f(w_1) = -(2r-5)$, $f(w_2) = 2r+3$.

Subcase 2.2: form = 3, 5 and 7, a pair sum labeling of (m,2)-kite is given in Figure 1. Hence the (m, 2)-kite is a pair sum graph for all $m \geq 3$.

Corollary 2.2: By deleting the pendant vertex in the (m,2)-kite of the proof of the Theorem 2.1 and by Theorem 1.1, the (m, 1)-kite is a pair sum graph except for m = 3.

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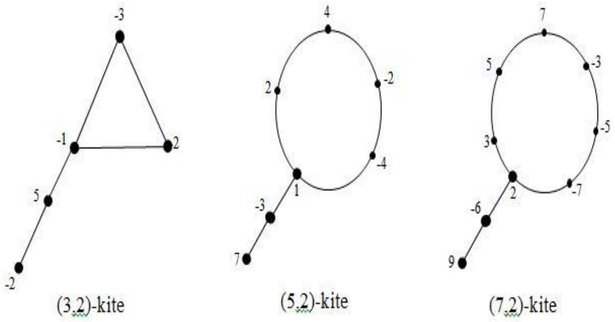

Figure 1

Theorem 2.3:The coconut tree CT(m, n) is a pair sum graph.
Proof: Let G(V,E) = CT(m, n). Then |V(G)| = m + n and |E(G)| = m + n-1
Case 1: $m \geq 5$

Subcase 1.1: m is even. Let m = 2r.

Then $V(G) = \{u_i, v_i / 1 \leq i \leq r\} \cup \{w_i / 1 \leq i \leq n\}$ and
$E(G) = \{u_r v_1, u_i u_{i+1}, v_i v_{i+1} / 1 \leq i \leq r-1\} \cup \{w_i / 1 \leq i \leq n\}$

Example 2.1:In Figure 2, pair sum labeling of (16,2)-kite and (17,2)-kite are illustrated.

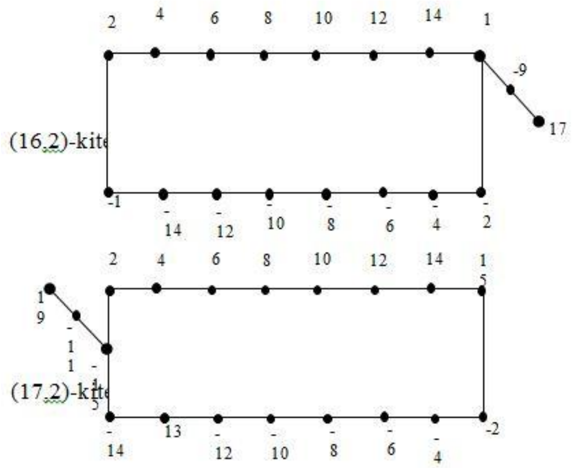

Figure 2: Pair sum labeling of (16,2)-kite and (17,2)-kite

Define $f : V \to \{\pm 1, \pm 2, \ldots, \pm(2r+n)\}$ as follows:
$f(u_1) = -3$, $f(u_r) = 1$, $f(v_r) = -1$
$f(u_i) = 2i$; $2 \leq i \leq r-1$
$f(v_i) = -f(u_i)$; $1 \leq i \leq r-1$

$$f(w_i) = \begin{cases} 7 & ; \quad i = 1 \\ -(2i+3) & ; \quad 2 \leq i \leq n \; \& \; i \text{ even} \\ (2i+3) & ; \quad 3 \leq i \leq n \; \& \; i \text{ odd} \end{cases}$$

Subcase 1.2: m is odd. Let m = 2r+1.
Then $V(G) = \{u_{r+1}, u_i, v_i / 1 \leq i \leq r\} \cup \{w_i / 1 \leq i \leq n\}$ and
$E(G) = \{u_i u_{i+1}, u_r v_1, v_r v_{r+1}, v_i v_{i+1} / 1 \leq i \leq r-1\} \cup \{v_{r+1} w_i / 1 \leq i \leq n\}$

Define $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm(2r+n+1)\}$ as follows:

$$f(u_i) = \begin{cases} 2(i+1) & ; \quad 1 \leq i \leq r-1 \\ 1 & ; \quad i = r. \end{cases}$$

$$f(v_i) = \begin{cases} -2i & ; \quad 1 \leq i \leq r \\ -1 & ; \quad i = r+1 \end{cases}$$

$$f(w_i) = \begin{cases} 2 & ; \quad i = 1 \\ 7 & ; \quad i = 2 \\ -(2i+1) & ; \quad 3 \leq i \leq n \end{cases}$$

Case 2: $m \leq 4$.

Subcase 2.1: m =1, then G = CT(1,n) is the star $K_{1,n}$. By Theorem 1.2 [2], G is a pair sum graph.

Subcase 2.2: m =2, then G = CT(2,n) is the star $K_{1,n+1}$. By Theorem 1.2 [2], it is a pair sum graph.

Subcase 2.3: m =3, then G = CT(3,n).
Let $V(G) = \{v_i / 1 \leq i \leq 3\} \cup \{w_i / 1 \leq i \leq n\}$ and $E(G) = \{v_1 v_2, v_2 v_3\} \cup \{v_3 w_i / 1 \leq i \leq n\}$

Label the vertices v_1, v_2 and v_3 by -3,2 and -1 respectively.

For $1 \leq i \leq n$, define $f(w_i) = \begin{cases} -(i+1) & ; \quad i \text{ odd} \\ (i+2) & ; \quad i \text{ even} \end{cases}$

Subcase 2.4: m=4 then G = CT(4,n).
$V(G) = \{v_i / 1 \leq i \leq 4\} \cup \{w_i / 1 \leq i \leq n$ and $E(G) = \{v_1 v_2, v_2 v_3, v_3 v_4\} \cup \{v_4 w_i / 1 \leq i \leq n\}$. Label the vertices v_1, v_2, v_3 and v_4 by -2,-4, 1 and 2 respectively.

For $1 \leq i \leq n$, define $f(w_i) = \begin{cases} 4 & ; i = 1 \\ -1 & ; i = 2 \\ -3 & ; i = 3 \\ i-1 & ; 4 \leq i \leq n \; \& \; i \text{ even} \\ -(i+2) & ; 5 \leq i \leq n \; \& \; i \text{ odd} \end{cases}$

Then f is a pair sum labeling.Hence the coconut tree CT(m, n) is a pair sum graph.

Example 2.2: In Figure 3 we give a pair sum labeling of CT(4,7).

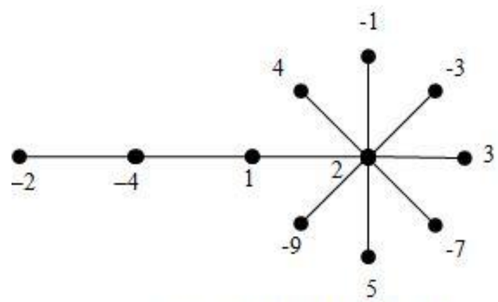

Figure. 3: Pair sum labeling CT(4,7)

Corollary 2. 4: For $r \geq 3$, the Y–tree Y_{r+1} is a pair sum graph.

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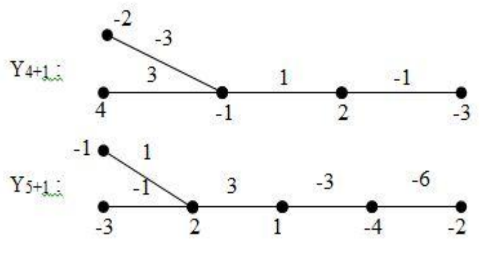

Figure 4: pair sum labeling of Y_{6+1}, Y_{4+1} and Y_{5+1}

Proof: We caneasily observe that$Y_{r+1} \cong CT(r-1,2)$ for $r \geq 3$. Hence by Theorem 2.1 , Y_{r+1} is a pair sum graph.

Example 2.3: In Figure, 4 a pair sum labeling of Y_{4+1} and Y_{5+1} are illustrated

Theorem 2.5: For m, n ≥1, Jelly fish graph J(m, n) is a pair sum graph.

Proof: Let G(V,E) = J(m, n). Then G has (m+n+4) vertices and (m+n+5) edges.

Let n ≥ m. Let V(G) = $V_1 \cup V_2$ where $V_1 = \{x, u, y, v\}$, $V_2 = \{ u_i, v_j ; 1 \leq i \leq m , 1 \leq i \leq n \}$ and $E = E_1 \cup E_2$,where $E_1 = \{xu, uy, yv, vx, xy \}$, $E_2 = \{uu_i, vv_j ; 1 \leq i \leq m , 1 \leq j \leq n \}$.

Define f : V → {±1, ±2, …, ±(m+n+4) } as follows:
Label the vertices x, u, y, v by -5, -1, 3 and 1 respectively.

Define

$$f(v_i) = \begin{cases} 5 & ; \quad i = 1 \\ 2(i-1) & ; \quad 2 \leq i \leq 4 \\ (i+2) & ; \quad 5 \leq i \leq m+1 \end{cases}$$

For $2 \leq j \leq n-m$

$$f(v_{m+j}) = \begin{cases} m+3+(j/2) & ; \quad 2 \leq j \leq m-n \,\&\, j\,\text{even} \\ -(m+5+(j-1)/2) & ; \quad 3 \leq j \leq m-n \,\&\, j\,\text{odd} \end{cases}$$

$f(u_i) = -f(v_{i+1})$; $1 \leq i \leq m$.
Then f is a pair sum labeling.Hence the Jelly fish graph J(m, n) is a pair sum graph.

Example 2.4:In Figure 5, a pair sum labeling for the Jelly fish graph J(4, 7) is exhibited.

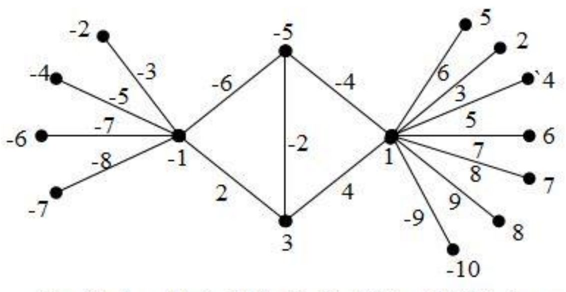

Figure 5: pair sum labeling for the Jelly fish J(4, 7) graph J(4, 7) pair sum

Next we will prove that the theta graph Θ ($l^{[m]}$) is a pair sum graph , for m even.

Theorem 2.6: For m even, the theta graph Θ ($l^{[m]}$) is a pair sum graph.

Proof: Let G(V,E) = Θ ($l^{[m]}$). Then |V(G)| = m(l- 1) +2 and |E(G)| = ml.
Let V(G) = $V_1 \cup V_2$ where

$$V_1 = \{u_{i,j}, v_{i,j} \mid 1 \leq i \leq m/2 \text{ and } 1 \leq j \leq l-1\},$$

$$V_2 = \{u, v\} \text{ and}$$

E(G) = $E_1 \cup E_2$ where

$$E_1 = \{u_{i,j} u_{i,j+1}, v_{i,j} v_{i,j+1} / 1 \leq i \leq m/2 \text{ and } 1 \leq j \leq l-2\},$$

$$E_2 = \{u\, u_{i,1}, u\, v_{i,l-1}, v\, v_{i,1}, v\, u_{i,l-1} / 1 \leq i \leq m/2 \}.$$

Define a map f : V(G) → {±1, ±2, …, ± m(l- 1) +2} as follows:
f(u) = 1, f(v) = – 1,
$f(u_{i,j}) = 2(li–i+ j – l+ 1)$; $1 \leq i \leq m/2$ & $1 \leq j \leq l-1$
$f(v_{i,j}) = f(u_{i,j})$); $1 \leq i \leq m/2$ & $1 \leq j \leq l-1$

Clearly f is a pair sum labeling.Hence the theta graph Θ ($l^{[m]}$) is a pair sum graph for m even.

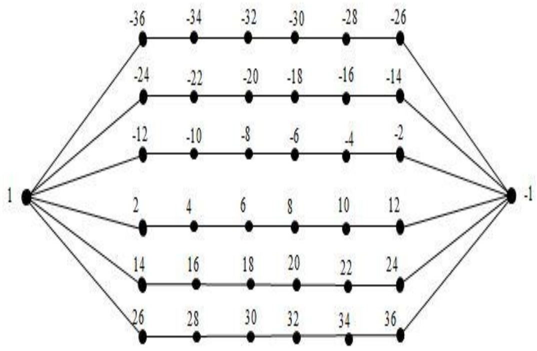

Figure 6: pair sum labeling for the theta graph Θ($7^{[6]}$)

Example 2.5:In Figure 6, a pair sum labeling for the theta graph Θ ($7^{[6]}$) is exhibited.

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