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DNA Computing Models.Springer. 2008. Domination in Permutation Graphs

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Abstract - If i, j belongs to a permutation on n symbols {1, 2, …, p} and i is less than j then there is an edge between i and j in the permutation graph if i appears after j. (i. e) inverse of i is greater than the inverse of j. So the line of i crosses the line of j in the permutation. So there is a one to one correspondence between crossing of lines in the permutation and the edges of the corresponding permutation graph. In this paper we found the conditions for a permutation to realize paths and cycles and also derived the domination number of permutation graph through the permutation. AMS Subject Classification (2010): 05C35, 05C69, 20B30.

Key Words: Permutation Graphs, Domination Number of a Permutation

I. DOMINATION IN PERMUTATION GRAPHS

Introduction: Adin and Roichman [1] introduced the concept of permutation graphs and Peter Keevash, Po-Shen Loh and Benny Sudakov [2] identified some permutation graphs with maximum number of edges. Charles J Colbourn, Lorna K.Stewart [3] characterized the connected domination and Steiner Trees under the Permutation graphs. In this paper we give an algorithm to find a minimal dominating set and so a set of all MDS of a permutation graph from the corresponding permutation. We proved the conditions for a complete bipartition and complete tripartition of a permutation graph and not all the graphs are permutation graphs.

Definition 1.1: Let π be a permutation on p symbols { $a_1,a_2,...,a_p$ }where image of a_i is a'_i. Then the PERMUTATION GRAPH G_π is given by (V_π , E_π) where $V_\pi = \{a_1,a_2,...,a_p\}$and $a_i a_j \in E_\pi$, if $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j)) < 0.$

Definition 1.2:The element a_i is said to DOMINATE a_j if their lines cross each other in π. The set of collection of elements of π whose lines cross all the lines of the elements $a_1,a_2,...,a_p$ in π is said to be a DOMINATING SET of π. $V = \{a_1,a_2,...,a_p\}$is always a dominating set.

Definition 1.3: The subset D of { $a_1,a_2,...,a_p$ } is said to be a MINIMAL DOMINATING SET of π if D-$\{a_i\}$ is not a dominating set of π for all $a_j \in$D.

Definition1.4: The DOMINATION NUMBER of a permutation π is the minimum cardinality of a set in MDS(π) and is denoted by γ(π).

Definition 1.5: A graph G is a PERMUTATION GRAPH if there exists a permutation π such that G_π = G. (i.e) a graph is a permutation graph if it is realizable by a permutation π. Otherwise it is not a permutation graph.

Definition 1.6: The NEIGHBOURHOOD of a_i in π is a set of all elements of π whose lines cross the line of a_i and is denoted by $N_\pi(a_i)$.

Theorem 1.7: The domination number of a permutation π is equal to the domination number of the corresponding permutation graph realized by π. (i.e) γ(π)= γ(G_π), the minimum cardinality of a minimal dominating set of G_π.

Proof: Let π be a permutation on a finite set A = { $a_1,a_2,...,a_p$ } and let G_π be the permutation graph, where V_π= A. Choose a_i & a_j in π whose lines cross each other. Then $\{a_i\}$or $\{a_j\}$will be in a dominating set, say D. Let D=$\{a_i\}$and let S = N_π(a_i). Let V_1= V- (D ∪ S). If V_1 = Φ, then D is a minimal dominating set of π. If not, either V_1 has elements whose lines cross the lines of π or has elements without crossing lines in π (trivial crossing). If all the elements of V_1 has trivial crossing then D_1=D ∪ V_1 is a minimal dominating set. If V_1 has elements whose lines cross the lines of elements of π then choose $a_r \in V_1$ whose line crosses the line of a_t in π. Then D_1= D ∪{a_r} or D_1=D ∪{a_t}. Let D_1= D ∪{a_r}. Then S_1 = S ∪$N_\pi(a_r)$, where $a_t \in N_\pi(a_r)$. Then V_2 = V_1- (D_1 ∪ S_1). If V_2 = Φ then D_1 is the minimal dominating set of π. If not, either V_2 has elements whose lines cross the lines of elements of π, or has elements which do not cross any of the lines of elements of π. As discussed earlier we arrive at a minimum dominating set D_2 = D_1 ∪ V_2 or D_2 = D_1 ∪ {a_s} and S_2 = S_1 ∪$N_\pi(a_s)$, where $a_r \in N_\pi(a_s)$. On continuing this process, after a finite stage k, we arrive at either V_k = Φ or V_k consisting of elements whose lines do not cross any of the lines of the elements of π. In both cases $D_k \cup V_k$ is a minimal dominating set of π. Thus all the minimal dominating sets MDS(π) can be established. The minimum cardinality of the set in MDS(π) is the domination number of π which is γ(π). There exists a 1-1 correspondence between the crossing of lines of elements of π and the edges of G_π. Hence γ(π)= γ(G_π). ■

II. DOMINATION IN COMPLETE BIPARTITE AND TRIPARTITE GRAPHS

Lemma 2.1: Let π be a permutation on p symbols $\{a_1,a_2,...,a_p\}$ such that $a_1 < a_2 < ..... < a_p$. Then the permutation graph is a

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complete graph if and only if the images of the elements are such that $a'_p < a'_{p-1} < \ldots. < a_1'$.

Proof: Let V_π= $\{a_1, a_2, ..., a_p\}$ such that $a_1 < a_2 < \ldots.. < a_p$ and $a'_p < a'_{p-1} < \ldots. < a_1'$. Let i < j . i,j = 1,2,…,p, i ≠ j. Then $a_i < a_j$.. Hence by hypothesis the images are in the reverse order. Therefore $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j)) < 0$. and so $a_i a_j \in E_\pi$. This is true if j < i also. Hence $a_i a_j \in E_\pi$, i ≠ j. So G_π is a complete graph. Conversely suppose that G_π is a complete graph. Let $G_\pi = K_p$. Let i < j. Hence $a_i < a_j$ and $\pi^{-1}(a_i) = k$ and $\pi^{-1}(a_j) = k'$. Then $a_i < a_j$ and as $a_i a_j \in E_\pi$ for all i≠j and so $\pi^{-1}(a_i) - \pi^{-1}(a_j) > 0$. (i.e) k > k'.

Therefore $\pi^{-1}(a_p) < \pi^{-1}(a_{p-1}) < \ldots < \pi^{-1}(a_1)$

(i.e) $a'_p < a'_{p-1} < \ldots. < a_1'$ ■

Theorem 2.2: Let π be a permutation on p symbols $\{a_1, a_2, ..., a_p\}$ such that $a_1 < a_2 < \ldots.. < a_p$. Then the permutation graph is a complete graph if and only if the images of the elements are such that $a'_p < a'_{p-1} < \ldots. < a_1'$. Then the domination number of π is 1.

Proof: Let π be a permutation on p symbols $\{a_1, a_2, ..., a_p\}$ such that $a_1 < a_2 < \ldots.. < a_p$ and $a'_p < a'_{p-1} < \ldots. < a_1'$. Then by Lemma 2.1, G_π corresponds to the complete graph K_p. Equivalently every line of an element in π crosses all the remaining lines of the elements of π. Therefore the domination number of π is 1.

Lemma 2.3: Let π be a permutation on S= $\{a_1, a_2, ..., a_p\}$ such that $a_1 < a_2 < \ldots.. < a_p$ and $a'_k < a'_{k+1} < \ldots.. < a'_p < a'_1 < a'_2 < \ldots\ldots < a'_{k-1}$ where k = 2,3,…,p. Then G_π is a complete bipartite graph.

Proof: Let V_π = S , $V_1 = \{a_k, a_{k+1}, \ldots\ldots, a_p\}$ & $V_2 = \{a_1, a_2, \ldots, a_{k-1}\}$ k = 2,3,…p. Let $a_i \in V_1$ and $a_j \in V_2$. (i.e) $a_j < a_i$. Then by hypothesis $a'_i < a'_j$.Therefore $\pi^{-1}(a_i) < \pi^{-1}(a_j)$ which implies $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j)) < 0$. Hence $a_i a_j \in E_\pi$, k ≤ i ≤ p, 1 ≤ j≤ k-1. Let $a_r, a_s \in V_1$. Assume that $a_r < a_s$. Then by the hypothesis, $\pi^{-1}(a_r) < \pi^{-1}(a_s)$ which implies that $(a_r - a_s)(\pi^{-1}(a_r) - \pi^{-1}(a_s)) > 0$. Hence $a_r a_s \notin E_\pi$. It is also true if $a_r > a_s$, $a_r a_s \notin E_\pi$. Therefore there is no edge among points of V_1. Similarly it can be seen that there is no edge among points of V_2. Hence G_π is a complete bipartite graph.

Theorem 2.4: Let π be a permutation on S= $\{a_1, a_2, ..., a_p\}$ such that $a_1 < a_2 < \ldots.. < a_p$ and $a'_k < a'_{k+1} < \ldots.. < a'_p < a'_1 < a'_2 < \ldots\ldots < a'_{k-1}$ where k = 2,3,…p. Then γ(π)=1, if k= 2 or p, and γ(π)= 2 if k = 3,4,..., p-1.

Proof: π is a permutation on S= $\{a_1, a_2, ..., a_p\}$ such that $a_1 < a_2 < \ldots.. < a_p$ and $a'_k < a'_{k+1} < \ldots.. < a'_p < a'_1 < a'_2 < \ldots\ldots < a'_{k-1}$ where k = 2,3,…p. Then by Lemma 2.3, G_π is a complete bipartite graph. If k = 2 and k = p then $G_\pi = K_{1,p-1}$ and hence γ(π) =1. If k = 3,4,...,p-1, by Lemma 2.3 γ(π)= 2.

Lemma 2.5:
Let π be a permutation on S= $\{a_1, a_2, ..., a_p\}$ where p is odd such that $a_1 < a_2 < \ldots.. < a_p$ and $a'_{\frac{p+1}{2}+k+1} < a'_{\frac{p+1}{2}+k+2} < \ldots.. < a'_p < a'_{\frac{p+1}{2}-k} < a'_{\frac{p+1}{2}-(k-1)} < \ldots.. < a'_{\frac{p+1}{2}} < a'_{\frac{p+1}{2}+1} < a'_{\frac{p+1}{2}+2} < \ldots.. < a'_{\frac{p+1}{2}+k} < a'_1 < a'_2 < \ldots\ldots < a'_{\frac{p+1}{2}-(k+1)}$ where k = 0,1,2,…, (p-3)/2. Then G_π is a complete tripartite graph.

Proof: Let V_π= $\{a_1, a_2, ..., a_p\}$. Let V_1= $\{a_1, a_2, \ldots\ldots, a_{\frac{p+1}{2}-(k+1)}\}$; V_2= $\{a_{\frac{p+1}{2}-k}, a_{\frac{p+1}{2}-(k-1)}, \ldots, a_{\frac{p+1}{2}}, a_{\frac{p+1}{2}+1}, \ldots, a_{\frac{p+1}{2}+k}\}$ and V_3= $\{a_{\frac{p+1}{2}+k+1}, a_{\frac{p+1}{2}+k+2}, \ldots, a_p\}$ where k = 0,1,2,3,…,(p-3)/2. Let $a_i \in V_1$ and $a_j \in V_2$. Then $a_i < a_j$. Then by hypothesis $a'_j < a'_i$. (i.e) $\pi^{-1}(a_j) < \pi^{-1}(a_i)$. Therefore $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j)) < 0$.. Hence $a_i a_j \in E_\pi$, $\forall i = 1, 2, \ldots., \frac{p+1}{2} - (k+1)$ and $j = \frac{p+1}{2} - k, \frac{p+1}{2} - (k-1), \ldots, \frac{p+1}{2}, \ldots, \frac{p+1}{2} + k$ where k = 0,1,2,…, (p-3)/2. Hence every vertex in V_1 is adjacent to all the vertices of V_2. Similarly it can be proved that every vertex in V_1 is adjacent to all the vertices of V_3 and every vertex in V_2 is adjacent to all the vertices of V_3. Now let us prove that there exists no edge among vertices of V_1. Let $a_r, a_s \in V_1$. Assume that $a_r < a_s$. Then by the hypothesis, $\pi^{-1}(a_r) < \pi^{-1}(a_s)$ which implies that $(a_r - a_s)(\pi^{-1}(a_r) - \pi^{-1}(a_s)) > 0$. Hence $a_r a_s \notin E_\pi$ .It is also true if $a_r > a_s$, $a_r a_s \notin E_\pi$. Therefore there is no edge among vertices of V_1. Similarly it can be seen that there is no edge among vertices of V_2 and among vertices of V_3. Hence G_π is a complete tripartite graph.

Lemma 2.6: Let π be a permutation on S= $\{a_1, a_2, ..., a_p\}$ where p is even such that $a_1 < a_2 < \ldots. < a_p$ and $a'_{\frac{p}{2}+k+1} < a'_{\frac{p}{2}+k+2} < \ldots.. < a'_p < a'_{\frac{p}{2}-(k-1)} < a'_{\frac{p}{2}-(k-2)} < \ldots.. < a'_{\frac{p}{2}} < a'_{\frac{p}{2}+1} < , \ldots, < \ldots < a'_{\frac{p}{2}+k} < a'_1 < a'_2 < \ldots\ldots < a'_{\frac{p}{2}-k}$ where k =1,2,…,(p/2)-1. Then G_π is a complete tripartite graph.

Proof: Let V_π= $\{a_1, a_2, ..., a_p\}$.Let V_1= $\{a_1, a_2, \ldots\ldots, a_{\frac{p}{2}-k}\}$; V_2= $\{a_{\frac{p}{2}-(k-1)}, \ldots, a_{\frac{p}{2}}, a_{\frac{p}{2}+1}, \ldots, a_{\frac{p}{2}+k}\}$ and V_3= $\{a_{\frac{p}{2}+(k+1)}, a_{\frac{p}{2}+(k+2)}, \ldots, a_p\}$ where k = 1,2,3,…,(p/2)-1. Let $a_i \in V_1$ and $a_j \in V_2$. Then $a_i < a_j$. By hypothesis $a'_j < a'_i$. (i.e) $\pi^{-1}(a_j) < \pi^{-1}(a_i)$ which implies $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j)) < 0$. .Hence $a_i a_j \in E_\pi$, $\forall i = 1, 2, \ldots., \frac{p}{2} - k$

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and $j=\frac{p}{2}-(k-1),\ldots,\frac{p}{2},\ldots,\frac{p}{2}+k$ where k = 1,2,3,…,(p/2)-1. Hence every vertex of V_1 is adjacent to all the vertices of V_2. Similarly it can be proved that every vertex of V_1 is adjacent to all the vertices of V_3 as well as between vertices of V_2 and V_3. Now let us prove that there exists no edge among vertices of V_1. Let $a_r, a_s \in V_1$. Assume that $a_r < a_s$. Then by the hypothesis, $\pi^{-1}(a_r) < \pi^{-1}(a_s)$ which implies that $(a_r-a_s)(\pi^{-1}(a_r)-\pi^{-1}(a_s))>0$. Hence $a_r a_s \notin E_\pi$. It is also true if $a_r > a_s$, $a_r a_s \notin E_\pi$. Therefore there is no edge among vertices of V_1. Similarly it can be seen that there is no edge among vertices of V_2 and among the vertices of V_3. Hence G_π is a complete tripartite graph. ■

Remark 2.7: Let π be a permutation on S= $\{a_1, a_2, \ldots, a_p\}$ such that $a_1 < a_2 < \ldots < a_p$. If π is expressed as a product of disjoint cycles such as $(a_1 a_{i+k})(a_2 a_{i+k+1})(a_3 a_{i+k+2}) \ldots (a_{i-k} a_p)$ where $i=\frac{p+1}{2}+1,\ k=1,2,\ldots,\frac{p-1}{2}$ for odd p and $i=\frac{p+2}{2},\ k=1,2,\ldots,\frac{p-2}{2}$ for even p, then G_π is a complete tripartite graph by Lemma 2.5 and Lemma 2.6.

Remark 2.8: The permutations following the pattern described in the Remark 2.7 always realizes a connected graph. Hence $1 \le \gamma(\pi) \le p/2$

Remark 2.9: The number of distinct permutations on p symbols yielding complete tripartite graphs is k = (p-1)/2 for odd p and k = (p-2)/2 for even p

Theorem 2.10: Let π be a permutation on S= $\{a_1, a_2, \ldots, a_p\}$ such that $a_1 < a_2 < \ldots < a_p$. If π is expressed as a product of disjoint cycles such as $(a_1 a_{i+k})(a_2 a_{i+k+1})(a_3 a_{i+k+2}) \ldots (a_{i-k} a_p)$ where $i=\frac{p+1}{2}+1,\ k=1,2,\ldots,\frac{p-1}{2}$ for odd p and $i=\frac{p+2}{2},\ k=1,2,\ldots,\frac{p-2}{2}$ for even p, then (i) γ(π) = 1 for k = 1 and odd p; (ii) γ(π) =1 if π = $(a_1 a_p)$; (iii) γ(π) = 2, otherwise.

Proof: Let π be a permutation on S= $\{a_1, a_2, \ldots, a_p\}$ such that $a_1 < a_2 < \ldots < a_p$. If π is expressed as a product of disjoint cycles such as $(a_1 a_{i+k})(a_2 a_{i+k+1})(a_3 a_{i+k+2}) \ldots (a_{i-k} a_p)$ where $i=\frac{p+1}{2}+1,\ k=1,2,\ldots,\frac{p-1}{2}$ for odd p and $i=\frac{p+2}{2},\ k=1,2,\ldots,\frac{p-2}{2}$ for even p, then π follows the pattern as described in Lemma 2.5 and Lemma 2.6 by the Remark 2.7. Hence (i) γ(π) = 1 for k = 1 and odd p; (ii) γ(π) = 1 if π = $(a_1 a_p)$; (iii) γ(π) = 2, otherwise.

III.REALIZABLE PERMUTATION GRAPHS

Lemma 3.1:
Let π be a permutation on S= $\{a_1, a_2, \ldots, a_p\}$ such that $a_1 < a_2 < \ldots < a_p$. and let (A) $a'_i = a_{i-2}$ odd i, 1< i < p, and $a'_j = a_{j+2}$, even j, 1≤ j < p-1, $a'_1 = a_2$ and $a'_{p-1} = a_p$ for odd p and $a'_p = a_{p-1}$ for even p (or) (B) $a'_i = a_{i+2}$, odd i, 1 < p, and $a'_j = a_{j-2}$,even j, 2< j ≤ p, a'₂= a_1 and $a'_p = a_{p-1}$ for odd p and $a'_{p-1} = a_p$ for even p. Then G_π is a path with p vertices.

Proof:
(A) Given $a'_i = a_{i-2}$ odd i, 1< i < p, and $a'_j = a_{j+2}$, even j, 1≤ j < p-1, $a'_1 = a_2$ and $a'_{p-1} = a_p$ for odd p and $a'_p = a_{p-1}$ for even p. Hence $\pi^{-1}(a_2)=a_1$; $\pi^{-1}(a_{j+2})=a_j$; 1≤ j < p-1; $\pi^{-1}(a_p)=a_{p-1}$ and $\pi^{-1}(a_{i-2})=a_i$ odd i , 1< i < p.

Case 1: Let m be odd.
Claim1: $a_m a_{m+1}, a_m a_{m+3} \in E_\pi$, 1 ≤ m < p. We know $a_m - a_{m+1} < 0$. $\pi^{-1}(a_m)=a_{m+2}$ and $\pi^{-1}(a_{m+1})=a_{m-1}$. Therefore $\pi^{-1}(a_m) - \pi^{-1}(a_{m+1})$ = a_{m+2} - a_{m-1} > 0 and hence ($a_m - a_{m+1}$)($\pi^{-1}(a_m) - \pi^{-1}(a_{m+1})$) < 0. So $a_m a_{m+1} \in E_\pi$. Similarly $a_m - a_{m+3} < 0$ and $\pi^{-1}(a_m) - \pi^{-1}(a_{m+3})$ = a_{m+2} - a_{m+1} > 0. Hence $a_m a_{m+3} \in E_\pi$,1 ≤ m < p.
Claim 2: $a_m a_{m+k} \notin E_\pi$ where k=4,5,6,…,p-m. Here $a_m - a_{m+k} < 0$, $\pi^{-1}(a_m)=a_{m+2}$ and $\pi^{-1}(a_{m+k})=a_{m+k+2}$ for even k and $\pi^{-1}(a_{m+k})=a_{m+k-2}$ for odd k. Therefore ($a_m - a_{m+k}$)($\pi^{-1}(a_m) - \pi^{-1}(a_{m+k})$) < 0. Hence $a_m a_{m+k} \notin E_\pi$ where k = 4,5,6,…,p-m.
Case 2: Let m be even.
m-1 and m-3 are odd and similar proof can be given to show that $a_m a_{m-1}, a_m a_{m-3} \in E_\pi$, 1 < m ≤ p − 1 and $a_m a_{m+k} \notin E_\pi$ where 1 < m < p, k =1,2,3,..., p-m.
Case 3: Let us prove that $a_p a_{p-2} \in E_\pi$, and $a_p a_{p-i} \notin E_\pi$, where i = 1,3,4,…,p-1; $a_1 a_2 \in E_\pi$ and $a_2 a_n \notin E_\pi$ 1 < n ≤ p . $a_p - a_{p-2} > 0$, $\pi^{-1}(a_p) - \pi^{-1}(a_{p-2})$ = a_{p-1} - a_p < 0. Therefore $(a_p - a_{p-2})(\pi^{-1}(a_p) - \pi^{-1}(a_{p-2})) < 0$. Hence $a_p a_{p-2} \in E_\pi$. We know that $a_p - a_{p-i} > 0$,i = 1,3,4,...,p-1. $\pi^{-1}(a_p) - \pi^{-1}(a_{p-i})$ = a_{p-1} - a_{p-k} >0, k=2,3,...,p-1. Hence $a_p a_{p-i} \notin E_\pi$, i=1,3,4,..,p-1. Similarly it can be proved that $a_1 a_2 \in E_\pi$ and $a_2 a_n \notin E_\pi$. Hence the permutation π given by $a'_i = a_{i-2}$ odd i, 1< i < p, and $a'_j = a_{j+2}$, even j, 1≤ j < p-1, $a'_1 = a_2$ and $a'_{p-1} = a_p$ for odd p and $a'_p = a_{p-1}$ for even p realizes a path $P_{p1} = \{a_2, a_1, a_4, a_3, \ldots a_p, a_{p-2}\}$. By the same argument as above it can be proved that π realizes the path $P_{p1} = \{a_2, a_1, a_4, a_3, \ldots a_p, a_{p-1}\}$ for even p.
(B) Similar proof can be set for the pattern given by π $a'_i = a_{i+2}$, odd i, 1≤ i < p, and $a'_j = a_{j-2}$,even j, 2< j ≤ p, a'₂= a_1 and $a'_p = a_{p-1}$ for odd p and $a'_{p-1} = a_p$ for even p. This pattern realizes the path $P_{p1} = \{a_1, a_3, a_2, a_5, \ldots a_p, a_{p-1}\}$ for odd p and $P_{p1} = \{a_1, a_3, a_2, a_5, \ldots a_{p-1}, a_{p-2}, a_p\}$ for even p.

Theorem 3.2: Let π be a permutation on S= $\{a_1, a_2, \ldots, a_p\}$ such that $a_1 < a_2 < \ldots < a_p$. and let (A) $a'_i = a_{i-2}$ odd i, 1< i < p, and $a'_j = a_{j+2}$, even j, 1≤ j < p-1, $a'_1 = a_2$ and $a'_{p-1} = a_p$

for odd p and $a'_p = a_{p-1}$ for even p (or) (B) $a'_i = a_{i+2}$, odd i, $1 \le i < p$, and $a'_j = a_{j-2}$,even j, $2 < j \le p$, a'$_2$= a_1 and $a'_p = a_{p-1}$ for odd p and $a'_{p-1} = a_p$ for even p. Then $\gamma(\pi) = \lceil p/3 \rceil$.

Proof: Let π be a permutation on S= $\{a_1, a_2, ..., a_p\}$ such that $a_1 < a_2 < .... < a_p$. and let (A) $a'_i = a_{i-2}$ odd i, $1 < i < p$, and $a'_j = a_{j+2}$, even j, $1 \le j < p-1$, $a'_1 = a_2$ and $a'_{p-1} = a_p$ for odd p and $a'_p = a_{p-1}$ for even p (or) (B) $a'_i = a_{i+2}$, odd i, $1 \le i < p$, and $a'_j = a_{j-2}$,even j, $2 < j \le p$, a'$_2$= a_1 and $a'_p = a_{p-1}$ for odd p and $a'_{p-1} = a_p$ for even p. Then by Lemma 3, G_π is a path with p vertices and hence $\gamma(\pi) = \lceil p/3 \rceil$.

Theorem 3.3: C_n, is not a permutation graph for any $n \ge 5$

Proof: When n = 3, according to Lemma 2.1 C_3 is a permutation graph. When n = 4 then by Lemma 2.3, C_4 is also a permutation graph. The permutations mentioned in the above theorem realize the path with p vertices. The vertices a_2 and a_p are adjacent to exactly one vertex each and other vertices are of degree 2 in case A and the vertices a_1 and a_{p-1} are adjacent to exactly one vertex each and other vertices are of degree 2 in case B. Therefore if a permutation has to realize a cycle, then the vertices a_2 and a_p in Case A, or a_1 and a_{p-1} in Case B must be of degree two along with the other vertices with degree two, which is not possible by the above theorem. Hence C_n, $n \ge 5$ are not permutation graphs.

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