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Extended Roman Domination Number of Honeycomb Networks

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Abstract - An extended Roman domination function on a graph G=(V,E) is a function satisfying the conditions that (i) every vertex u for which f(u) is either 0 or 1 is adjacent to at least one vertex v for which f(v) =3,(ii) if u and v are two adjacent vertices and if f(u)=0 then f(v)≠0. The weight of an extended Roman domination function is the value $f(V) = \sum_{u \in V} f(u)$.The minimum weight of an extended Roman domination function on graph G is called the extended Roman domination number of G, denoted by $\gamma_{R_\epsilon}(G)$. In this paper we study this variant of domination for honeycomb networks.

Keywords:Domination, Extended Roman domination, Extended Roman domination number, Honeycomb networks.

I. INTRODUCTION

Given a graph *G=(V; E)* of order *n*. A set of vertices *S* in *G* is a dominating set, if for every vertex *v∈V/S* there exist a vertex *u* *∈S* such that *v* is adjacent to *u*. The domination number of *G*, denoted by *γ(G)*, is the minimum cardinality of a dominating set of *G*.[4]

As the study of dominations in graph increased the research led to different types of dominations in graphs such as total domination in graphs, weak domination, total restrained domination, power domination etc. which in turn led to the concept of domination numbers in graphs. Research was also carried out in finding upper bounds for different domination numbers. In this context also came the concept of Roman domination number. This concept was first defined by Cockayne et al. The concept is taken from the ancient Rome. Rome was the super power in beginning of the first century. It needed to keep its dominating power at the same time defend its empire. The Emperor Constantine decreed that his empire should be defended from the attacks of the enemy with minimum cost. Keeping army all around the boarder of the empire was expensive. So he designs a method which allowed him to defend his empire with minimum cost. According to it he keeps soldiers at different locations and at some locations no soldiers but at those locations soldiers could be sent at the time of enemy attack. This same concept was studied in graph theory by different mathematicians, which came to be known as Defending the Roman Empire and then the concept Roman Domination Number emerged. We extend this concept to Extended Roman Domination Number by adding some more conditions. An extended Roman domination function on a graph *G=(V,E)* is a function $f: V \to \{0,1,2,3\}$ satisfying the conditions that (i) every vertex *u* for which *f(u)* is either 0 or 1 is adjacent to at least one vertex *v* for which *f(v) =3*,(ii) if *u* and *v* are two adjacent vertices and if *f(u)=0* then *f(v)≠0*[1]. In this paper, we study a variant of the domination number called Extended Roman domination number for honeycomb networks.For notation and graph theory terminology in general we follow [4].

II. PROPERTIES OF EXTENDED ROMAN DOMINATION SETS

For a graph *G=(V,E),* let $f: V \to \{0,1,2,3\}$ and let (V_0, V_1, V_2, V_3)be the ordered partition of *V* induced by *f*, where $V_i = \{v \in V | f(v) = i\}$and $|V_i| = n_i$, for *i*=0,1,2,3. Note that there exists a 1-1 correspondence between the functions $f: V \to \{0,1,2,3\}$ and the ordered partitions $(V_0, V_1, V_2, V_3)$of V. Thus, we will write $f = (V_0, V_1, V_2, V_3)$.

A function $f = (V_0, V_1, V_2, V_3)$ is an extended Roman domination function if, (i) $V_3 \succ V_0 \cup V_1$, where $\succ$ means that the set V_3 dominates the set $V_0 \cup V_1$, i.e. $V_0 \cup V_1 \subseteq N[\ V_3]$and (ii) G $(V_0) = \overline{K_{n_0}}$, where G (V_0) is the subgraph induced by V_0. The weight of f is $f(V) = \sum_{u \in V} f(u) = 3n_3 + 2n_2 + n_1$. We say a function $f = (V_0, V_1, V_2, V_3)$ is a γ_{R_ϵ}- *function* if it is an extended Roman domination function and$f(V) = \gamma_{R_\epsilon}(G)$.[1]

Proposition 2.1[1]: For any graph G of order n, $\gamma_{R_\epsilon}(G) = 2\gamma(G)$ if and only if $G = \overline{K}_n$

Proposition 2.2[1]: Let $f = (V_0, V_1, V_2, V_3)$ be any γ_{R_ϵ}- *function* .Then

a) G (V_2) the subgraph induced by V_2 has max degree 1.
b) $V_2 \cup V_3$is the dominating set for the graph G.
c) V_3 dominates$V_0 \cup V_1$.
d) The subgraph induced by $V_0 \cup V_3$ is either a tree or it is a disconnected graph whose each component is a tree.
e) V_3 is the dominating set for G$(V_0 \cup V_1 \cup V_3)$.
f) Let H = G$(V_0 \cup V_1 \cup V_3)$ then each vertex$v \in V_3$ has atleast two H-pn's.(for n>2)

Theorem 2.1[1]:
$\frac{2n}{\Delta+1} \leq f(V) \leq 2n,$ where $f(V)$ is the weight of *f*i.e. $f(V) = \sum_{u \in V} f(u)$

Proposition 2.3[1]: For any graph G of order n, $\gamma_{R_\epsilon}(G) = 3$ if and only if G $= K_{1,n-1}$.

Theorem 2.4[1]: Let Gbe a graph. For any pair of non-adjacent vertices {x,y} in G, $\gamma_{R_E}(G+xy) \leq \gamma_{R_E}(G)+1$

2.1. Specific Values Of Extended Roman Domination Numbers

Proposition 2.1.1:[1]For the classes of path P_n,

$$\gamma_{R_E}(P_n) = \left\lceil \frac{4n}{3} \right\rceil \text{ if } n<3 \text{ and if } n\geq 3 \text{ then } \gamma_{R_E}(P_n) \leq \left\lfloor \frac{4n-1}{3} \right\rfloor$$

Proposition 2.1.2:[1]For the classes of cycles $C_n, n \geq 3$

$$\gamma_{R_E}(C_n) \leq \begin{cases} \left\lfloor \frac{4n-1}{3} \right\rfloor & \text{if } n \equiv 1 \bmod 3 \\ \left\lceil \frac{4n}{3} \right\rceil & \text{otherwise} \end{cases}$$

Proposition 2.1.3:[1]For the complete graph K_n, $\gamma_{R_E}(K_n) = n+1.$

Proposition 2.1.4:[1]For $m < n, \gamma_{R_E}(K_{m,n}) = \begin{cases} 2m+1 & \text{if } m \leq 3 \\ m+5 & \text{if } m > 3 \end{cases}$

Proposition 2.1.5 :[1]For the $m \times n$ Cylinder $CY_{m,n}$ $m,n>1$,

$$\gamma_{R_E}(CY_{m,n}) \leq \begin{cases} 2\left\lceil \frac{m}{2} \right\rceil \left\lceil \frac{n}{2} \right\rceil + \left\lfloor \frac{m}{2} \right\rfloor \left(3\left\lfloor \frac{n}{2} \right\rfloor + 1 \right) & \text{if } n \text{ is odd} \\ 2\left\lceil \frac{m}{2} \right\rceil \left\lceil \frac{n}{2} \right\rceil + 3\left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{n}{2} \right\rfloor & \text{otherwise} \end{cases}$$

III. UPPER BOUND FOR EXTENDED ROMAN DOMINATION NUMBER FOR HONEYCOMB NETWORK

Honeycomb networks are built recursively using hexagonal tessellations. The honeycomb network $HC(1)$ is a hexagon. The honeycomb network $HC(2)$ is obtained by adding six hexagons to the boundary edges of $HC(1)$. Inductively, honeycomb network $HC(n)$ is obtained from $HC(n-1)$ by adding a layer of hexagons around the boundary of $HC(n-1)$. The parameter n of $HC(n)$ is determined as the number of hexagons between the centre and boundary of $HC(n)$. The number of vertices and edges of $HC(n)$ are $6n^2$ and $9n^2-3n$ respectively. A honeycomb network HC(3) is shown in Figure1.In Graph Theory to study the honeycomb network we use brick structure of the honeycomb networks. Brick structure is obtained by shrinking one of the upper and lower vertices in the straight lines. Thus in brick representation also there are equal number of vertices and edges. A brick representation of *HC(3)* is shown in Figure 2. The application of Honeycomb network is very vast; it is applied in different networking's such as all-to-all broadcasting, in cellular services, in computer networking. It is also used in chemistry to represent the structures of different compounds.The telecommunication industry has always been a host for a wide variety of optimization problems. In recent years, extensive research has led to the rapid development of numerous mobile computing devices that are diverse and technologically intensive.

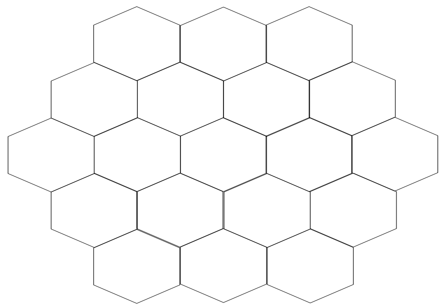

Figure1: Honeycomb network of dimension 3

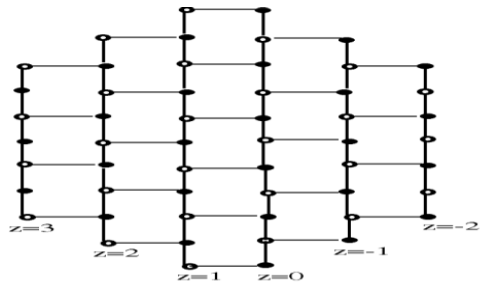

Figure 2: Brick representation of HC(3)

This has in turn, posed a variety of challenges to the scientific and engineering research community to provide the necessary algorithms and protocols to utilize these systems and at times, overcome their drawbacks. Wireless networks such as satellite networks, radio networks, sensor networks, cellular networks, ad hoc networks and other mobile network where honeycomb networks is used extensively.

Theorem 4.1: The extended Roman Domination number for any n dimensional Honeycomb Networks is HC(n) is given by,

Proof: The proof is given by construction. There are total $\boldsymbol{6n^2}$vertices in any n dimensional Honeycomb Network.The central block of the honeycomb network consists of *2(4n-1)* vertices.

$$\gamma_{R_E}(HC(n)) \leq \begin{cases} \frac{(5n-1)(3n+1)}{2}, & \text{when } n \text{ is odd.} \\ \frac{15n^2}{2}, & \text{when } n \text{ is even.} \end{cases} \quad \square$$

We label the vertices as follows:

On the upper half of the central block we label (0,3) alternately. Then we label the above row with (0,2) alternatively as shown in the Figure 3. And then we continue with (0,3) the above row and next above row with (0,2). We continue in this pattern for the rest of the rows above.Next we label the lower part of the middle vertices with (1,3) alternatively. Next row below that we label the vertices with(0,2) and below with (0,3) and continue in this pattern till we label all the vertices. When n is even integer, the number of vertices which are labeled with 1 in the HC(n) network is 2n, i.e. $|V_1|$ =2n.The number of vertices which are labeled with 2 in the HC(n) network is $\frac{n}{2}(3n-2)$ i.e. $|V_2| = \frac{n}{2}(3n-2)$ .The number of vertices which are labled with 3 in HC(n) network is $\frac{3n^2}{2}$. i.e. $|V_3| = \frac{3n^2}{2}$. And the rest of the vertices are labled with 0.

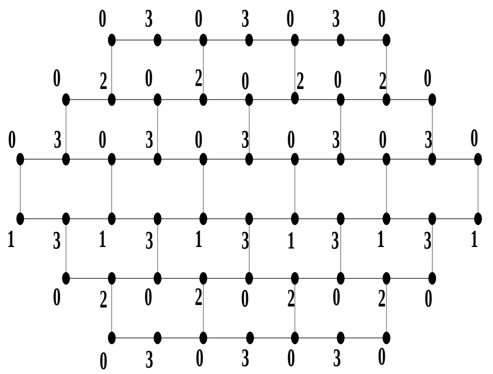

Figure 3. Extended Roman domination function for *HC(3)*.

When *n* is odd integer, the number of vertices which are labled with 1 in the *HC(n)* network is 2*n* i.e. $|V_1| = 2n$.The number of vertices which are labled with 2 in the *HC(n)* network is $\frac{(n-1)(3n-1)}{2}$ i.e. $|V_2| = \frac{(n-1)(3n-1)}{2}$. The number of vertices which are labled with 3 in *HC(n)* network is $\frac{(n+1)(3n-1)}{2}$ i.e. $|V_3| = \frac{(n+1)(3n-1)}{2}$. And rest of the vertices are labled with 0.

Therefore when n is even the weightof the extended Roman domination function of *HC(n)* is given by, $|V_1| + 2|V_2| + 3|V_3| = 2n + 2\left(\frac{n(3n-2)}{2}\right) + 3\left(\frac{3n^2}{2}\right) = \frac{15n^2}{2}$ .The weight of the extended Roman domination function of *HC(n)* when *n* is odd is given by

, $|V_1| + 2|V_2| + 3|V_3| = 2n + 2\left(\frac{(n-1)(3n-1)}{2}\right) + 3\left(\frac{(n+1)(3n-1)}{2}\right) = \frac{(5n-1)(3n+1)}{2}$

Hence

$$\gamma_{R_E}(HN(n)) \leq \begin{cases} \frac{(5n-1)(3n+1)}{2}, & when\ n\ is\ odd. \\ \frac{15n^2}{2}, & when\ n\ is\ even. \end{cases}$$

IV. CONCLUSION

In this paper we have studied some properties of the extended Roman domination number of a graph and have obtained the upper bound for n dimensional Honeycomb network. This work could be further extended to other classes of graphs.

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