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Modular Super Vertex Magic Total Labelling

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Abstract - Let G be a finite simple graph with |V| = p and |E| = q. We extend the idea of super vertex magic total labeling to modular super vertex magic labeling. In this paper we give some properties of modular super vertex magic total labellings. Here we find some families of graphs that admit this labeling and also some other graph which do not possess this property.

Keywords: Modular super vertex magic total labeling, Super vertex magic total labeling, Vertex magic total labeling.

I. INTRODUCTION

In this paper consider only finite simple graphs. The set vertices and edges of a graph *G* will be denoted by *V (G)* and *E(G)* respectively $p = |V|$ and $q = |E|$. The set of vertices adjacent to a vertex *u* of *G* is denoted by *N(u).* The vertex magic total labeling (VMTL) was introduced in [4]. This is an assignment of integers from 1 to $p+q$ to the vertices and edges of *G*,so that at each vertex, the vertex label and the labels on the edges incident at that vertex, adds to a fixed constant. $f(u)+\sum_{u\in N(u)} f(uv)=k$ Where the sums runs over all vertices *v* adjacent to *u*. MacDougall, Miller and Sugeng [7] called a vertex magic total labeling is super if *f(V(G)) = (1, 2, 3,… p)*. i.e. the smallest labels are assigned to the vertices and *f(E) = {|V|+1,|V|+2,…|V|+|E|}*. Swaminathan and Jeyanthi [9] called a vertex magic total labeling is super if *f(E(G)) = (1, 2, 3, … q)*. But Marimuthu and Balakrishnan [2] called the above type of labeling as *E*-super vertex magic total labeling. To avoid confusion in this paper we use, in super vertex magic total labeling the smallest labels are assigned to the vertices and the *E*-super vertex magic total labeling the smallest labels are assigned to the edges. In [7] they proved that an *r*-regular graph of order *p* has a super vertex magic total labeling then *p* and *r* have opposite parity and if $p \equiv 0 (mod\ 8)$ then $q \equiv 0 (mod\ 4)$. If p ≡ 4(mod 8) then q ≡ 2(mod 4). The cycle C_n has a super vertex magic total labeling if and only if *n* is odd. They also conjectured that if $n \equiv 0(mod\ 4);\ n > 4$, then $K_n$hasa super vertex magic total labeling. But this conjecture was proved by J.Gomez in [4] also tree, wheel, fan, ladder, or friendship graph has no super vertex magic total labeling. If *G* has a vertex of degree one, then *G* is not super vertex magic total labeling. Swaminathan and Jeyanthi [9] showed that the path P_n has a *E*-super vertex magic total labeling if and only if *n* is odd and if and only if *n = 2*. $mC_n$is*E*-super vertex magic total labeling if and only if *m* and *n* are odd. Marimuthu and Balakrishnan [2] proved that, for a connected graph *G* and *G* has a E-super vertex magic total labeling with magic constant *k*then $k \geq \frac{5p-3}{2}$. Also proved for a *(p, q)* graph, with even *p* and *q = p -1* or *p*, then the graph is not *E*-super vertex magic total labeling. Generalized Petersen graph *P(n,m)* is not *E*-super vertex magic total labeling if *n* is odd. They also discussed about the *E*-super vertex magicness of *m* connected graph $H_{m,n}$. A graph with odd order can be decomposed into two Hamiltonian cycles, then *G* is *E*-super vertex magic total labeling. A graph *G* can be decomposed into two spanning sub graphs G_1 and G_2 where G_1 is E-super vertex magic and G_2 is magic and regular then *GE*-super vertex magic. Also they proved as the two spanning sub graphs are *E*-super vertex magic and one is regular then the graph will *E*-super vertex magic. Many kinds of labeling have been studied and an excellent survey of graph labeling can be found in [1].

II. MODULAR SUPER VERTEX MAGIC TOTAL LABELING

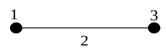
In this section we introduce a new concept standing on the basis of super vertex magic total labelling.

Definition 2.1 A one – one map *f* from *V* ∪ *E* of a graph *G* onto the integers *{1,2,3…p+q}* is a modular vertex magic labeling if there exist a constant *k* such that $f(x)+\sum f(xy)=k(\text{mod } p), \forall x\in V, 0\leq k < p-1.$

Definition 2.2 A graph which admits modular super vertex magic labelling then it is called modular super vertex magic graph.

The following graph does not have vertex magic total labeling, but it has a modular vertex magic labelling.

Example



Theorem 2.1 Let *G*be a graph. If *G* has a vertex magic total labeling then it has a modular super vertex magic labelling. But converse need not be true.

Definition 2.3 A vertex magic total labeling is called a modular super vertex magic total labeling if *f(V(G)) = {1,2,3…p}*

Definition 2.4 A graph which admits modular super vertex magic total labeling then it is called modular super vertex magic total graph.

heorem 2.2 Let *G*be a graph with *p* vertices and q edges. If *G* has a modular super vertex magic total labeling then p divides $\frac{p(p+1)}{2}+q(q+1)$.

Proof: Let *f*be a modular super vertex magic total labeling and let $wt(x)=f(x)+\sum f(xy), \forall x\in V.$

Then $wt(x) \equiv k(\ mod\ p)$

So $\sum x\in V, wt(x)\equiv pk\equiv 0(\text{mod } p)$

This implies, $\sum_{x\in V} f(x)+2\sum_{e\in E} f(e)\equiv 0(\text{mod } p)$

$$\frac{p(p+1)}{2}+2\left[\frac{(p+q)(p+q+1)}{2}-\frac{p(p+1)}{2}\right]\equiv 0(\text{mod } p)$$

$$\frac{p(p+1)}{2}+q(2p+q+1)\equiv 0(\text{mod } p)$$

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So $\frac{p(p+1)}{2}+q(q+1)\equiv 0(\bmod p)$

Therefore, p divides $\frac{p(p+1)}{2}+q(q+1)$ ■

Theorem 2.3 Let Gbe a graph with p vertices and q edges. If p is even and $q = p-1$or p.
Then G has no modular super vertex magic total labeling.
Proof: Consider $q = p$
By theorem 2.2, clearly p divides $p(p+1)$
Since p is even, p does not divides $\frac{p(p+1)}{2}$

Therefore,p does not divide $\frac{p(p+1)}{2}+p(p+1)$

∴G has no modular super vertex magic total labeling.
Same thing happens when $q = p-1$■
Corollary 2.1 A star graph with even number of vertices has no modular super vertex magic total labelling.
Proof: Let G be a star graph has p vertices and p is even. Then it has $p-1$ edges.
Therefore by theorem 2.3, star graph with even vertices has no modular super vertex total magic labelling.■
Corollary 2.2 A cycle C_n with even number of vertices has no modular super vertex magic total labelling.
Corollary 2.3 A path $P_n$with even number of vertices has no modular super vertex magic total labelling.
Corollary 2.4 A tree with even number of vertices has no modular super vertex magic total labelling.
Lemma 2.1 IfG(p,q) has modular super vertex magic total labelling and p/q then p is not even.

Theorem 2.4 $K_{m,m}$ has no modular super vertex magic total labeling.

Proof: For $K_{m,m}$, number of vertices $p = 2m$ and number of edges $q = m^2$.

By theorem 2.1, p divides $\frac{p(p+1)}{2}+q(q+1)$

⇒$2m$ divides $\frac{2m(2m+1)}{2}+m^2(m^2+1)$

⇒$2m$ divides $m(2m+1) + m^2(m^2+1)$
⇒$2m$ divides $m(m^3+3m+1)$
⇒$2m$divides m^3+3m+1
No such m will exist.
∴$K_{m,m}$has no modular super vertex magic total labeling.■

Theorem 2.5 IfG is a star graph with odd number of vertices then there exist a modular super vertex magic total labeling with $k = 1$.
Proof: Let the star graph G has p vertices where$p$is odd. Let v_1 be the root and $v_2,v_3…v_p$ be the other vertices.
Let $v_1v_i = e_{i-1}$, $I = 2,3…p$
Define $f(v_i) = i$, $1,2,3…p$ and $f(e_j) = 2p-j$, $j = 1,2,3…p-1$.
Then $wt(V_i) = f(v_i) + f(e_{i-1})$, $i = 2,3…p-1$
$= i + 2p -(i-1)$
$= 2p+1\equiv 1(mod\ p)$
$wt(V_1) = f(v_1) + \sum f(e_i)$
$= 1 + [(p+1) + (p+2) + … +(2p-1)]$
$= 1 + p(p-1) + (1 +2 +… + (p-1))$
$= 1 + p(p-1) + \frac{p(p-1)}{2}$

Since p is odd, ⇒ p/ $\frac{p(p-1)}{2}$

∴$wt(V_i) \equiv 1(mod\ p)$■
Theorem 2.6 Let$G$ be a graph with p vertices and q edges. If G has a E-super vertex total labelling, and $(deg(u)+1)(q+1) \equiv r(mod\ p)$. Then G has modular super vertex magic total labelling. The converse need not be true.
Proof: Since G has a E-super vertex magic labelling, there exist a function g from $V(G)\cup E(G)$ to $1,2,3…,p+q$ such that $g(E(G)) = \{1,2,3…, q\}$ and $g(u)+\sum g(uv)=k$.
Define $f(u) = p+q+1 - g(u)$ for every vertex $u \in V(G)$ and $f(uv) = p + q + 1 - g(uv)$for every edge $uv \in E(G)$.
Since $g(E(G)) = \{1,2,3…q\}$ it is clear that $f(V(G)) = \{1,2,3…p\}$
Now, $wt_g(u)=g(u)+\sum_{u\in N(u)} g(uv)=k$

Therefore,
$$wt_f(u) = f(u)+\sum_{u\in N(u)} f(uv)$$
$= (p + q +1 - g(u)) + \sum(p+q+1-g(uv))$
$= (deg(u) + 1)(p + q +1) - wt_g(u)$
$= (deg(u) + 1)(p + q + 1) - k$
$= (deg(u) + 1)(q + 1) - k(mod\ p)$
$= r - k(mod\ p)$
∴f is a modular super vertex magic total labelling.A star graph with odd number of vertices p has a modular vertex total labelling and it satisfies the condition $(deg(u) + 1)(q + 1) \equiv r(mod\ p)$ with $r = 0$.By [2] a star graph S_n is E-super vertex magic if and only if $S_n = P_2$.
∴ In general a star graph with odd number of vertices has no E-super vertex magic labelling. ■

REFERENCES

[1] D Antony Xavier, DeeniC.J ,*"Super vertex magic and E – super vertex magic total labelling"*,Proc.of ICAMTCS, 67-74 ,2013.
[2] Marimuthu, M.Balakrishnan, *"E-Super vertex Magic Labelling's of Graphs"*, Discrete Applied Mathematics,160, 1766-1774 , 2012.
[3] .A. Gallian, *"A dynamic survey of graph labelling"*,the Electronic Journal of Combinatorics 18, 2011.
[4] A.A.G.Ngurah,A.N.M.Salman,L.Susilowati, *"H-Super Magic Labelling's of Graphs"*, Discrete Mathematics,310,1293-1300, 2010.
[5] J.Gomez, *"Two new methods to obtain super vertex magic total labelling of graphs"*, Discrete Math.308,3361-3372, 2008.
[6] J.Gomez,*"Solution of the conjecture :if n ≡ 0(mod4); n > 4 then Kn has super vertex magic total labelling"*, Discrete Math.307,2525-2534, 2006.
[7] C.Balbuena,E.Barker,K.C.Das,Y.Lin,M.Miller,J.Ryan,Slamin,K.Sugeng, M.Tkac,*"On the Degrees of a Strongly Vertex Magic Graph"*,Discrete Mathematics,306,539-551, 2006.
[8] V. Swaminathan, P. Jayanthi, *"On Super vertex Magic Labelling"*, J.DiscreteMath.Science and Cryptography, 8, 217-224,2004.
[9] J.A MacDougall, M. Miller, K.A Sugeng,*" Super Vertex Magic Total Labelling of Graphs"*, In Proc. of the 15th Australian workshop on Combinatorial Algorithms pp. 222-229, 2004.
[10] V. Swaminathan, P. Jayanthi,*" Super vertex Magic Labelling"*, Indian Journal of pure Applied Math 34(6), 935-939,2003.
[11] J.A MacDougall, M. Miller, Slamin, W.D. Willis, *"Vertex Magic Total Labelling of Graphs"*, util. Math, 61, 3-21, 2002.
[12] J.A.Bondy, U.S.R. Murty, *Graph Theory with Applications*, North Holland, New York.Amsterdam.Oxford,1976.