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On Edge Antimagic Total Labeling and Arithmetic Progression

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Abstract- An edge magic total labeling of graph with p vertices and q edges is a bijection from the set of vertices and edges to 1, 2, 3,…,|p|+|q| such that for every edge the sum of the label of the edges and the label of its two end vertices are constant . And if the sum is distinct it is said to be an edge antimagic total labeling. In this paper, we exhibit edge antimagic total labeling on graph structure for various arithmetic progression.

Index Terms - graph, labeling, function, arithmetic progression, magic labeling.

I. INTRODUCTION

A graph G = (V,E) with p vertices and q edges is called total edge magic if there is a bijection function f: VUE → {1,2,3,…,p+q} such that for any edge uv in E we have a constant K with f(u)+f(v)+f(uv) = K . A graph G = (V,E) with p vertices and q edges is called total edge antimagic if there is a bijection function f: VUE→ {1,2,3,…,p+q} such that for any edge uv in E we have f(u)+f(v)+f(uv) is distinct [1,5]. Many people worked on magic and anti magic labeling on the graph [3]. In this paper, we will study that if we have some arithmetic progression, then it can exhibit as an edge antimagic total labeling on graph structure. The minimum number we can start with it is 6 because the minimum sum of first three numbers is 6 (1+2+3), if the jump is 6 then we can get path graph as in the next example (fig 1).

Example 1:If S_n denotes the edge counts with the jump n, then the edge counts of the path P_6 is given in the form of an arithmetic progression S_6= {6, 12, 18, 24, 30}. In general we can say S_6 = { m,m+a,m+2a,…,m+(n-2)a } such that m=6,a=6, start from 6 with jump 6 is given in the fig 1 .

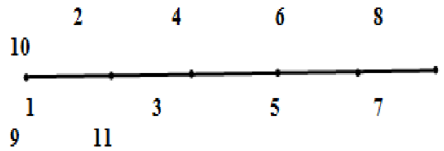

Fig 1: Edge antimagic total labeling of the path P_6.

The next examples take different cases of the arithmetic progression with different jump and different number can we start on it.

Example 2: The edge counts S_1= {14, 15, 16, 17, 18}is of the from of an arithmetic progression { m,m+a,m+2a,…,m+(n-2)a } such that m≥8 , m≡0(mod 2) ,a=1, n=(m-2)⁄2 start from 14 with jump 1 is given in fig 2.

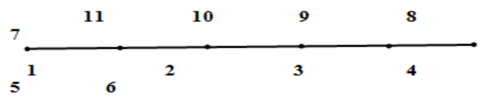

Fig 2: Edge antimagic total labeling of the path P_6.

II. ARITHMETIC PROGRESSION ON PATH GRAPH

We provide algorithms for labeling for our results that makes the flow of the proof simple.

Algorithm-2.3
Input: The arithmetic progression of the form
S_1 = { m,m+a,m+2a,…,m+(n-2)a} such that m≥8 , m≡0(mod2) ,a=1.
Output: Construction and edge antimagic total labeling for this arithmetic progression as a path graph.
Begin

Step 1: The vertex set V = {v_i: 1≤ i≤ n} and the edge set E = {v_iv_{i+1}: 1 ≤i ≤ n-1}
Step 2: f (v_i) = i, 1 ≤i ≤ n
Step 3: f (v_iv_{i+1}) = 2n-i 1 ≤i≤ n-1
End

Theorem 2.2 : The path P_n admit edge antimagic total labeling in the form of an arithmetic progression S_1 ={ m,m+a,m+2a,…,m+(n-2)a} when m≥8 , m≡0(mod2) ,a=1, n=(m-2)⁄2 .

Proof : The above algorithm 2.1 construct the arithmetic progression of the form
S_1 = { m,m+a,m+2a,…,m+(n-2)a } with the path graph P_n such that n=(m-2)⁄2
Let f: VUE→ {1, 2, 3,…, 2n-1} be the bijective function defined as in the step 2

Then for all 1 ≤i≤ n-1
$$f(v_i) + f(v_{i+1})+f(v_iv_{i+1}) = i+i+1+2n-i$$
$$= 2n+i+1 = (2n+1) +i$$
$$= (2((m-2)/2) +1) +i$$

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$= (m-1)+i$

Now using values of i we get edge counts as {m, m+1, m+2,…, m+n-2}Which concludes that there exist an edge antimagic total labeling which is path graph for each arithmetic progression of the form S_1(jump 1).

Example 3: The edge counts S_2= {14, 16, 18, 20, 22} from of an arithmetic progression {m,m+a,m+2a,…,m+(n-2)a} such that m≥8 , m≡0(mod2) ,a=2, n=(m-2)⁄2, start from 14 with jump 2 is given in fig 3.

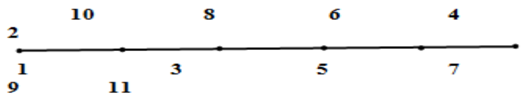

Fig 3

Algorithm- 2.4

Input: the arithmetic progression of the form

S_2 = { m,m+a,m+2a,…,m+(n-2)a } such that m≥8 , m≡0(mod2) ,a=2.

Output: Construction and edge antimagic total labeling for this arithmetic progression as a path graph.

Begin

Step 1: The vertex set V = {v_i : 1≤ i≤ n} and the edge set

E = {v_iv_{i+1} : 1 ≤i ≤ n-1 }

Step 2: f (v_i) = 2i-1, 1 ≤i ≤ n

Step 3: f (v_iv_{i+1}) = 2n-2i 1 ≤i≤ n-1

End

Theorem 2.5 : The path P_n admit edge antimagic total labeling in the form of an arithmetic progression S_2 ={ m,m+a,m+2a,…,m+(n-2)a} when m≥8 , m≡0(mod2) ,a=2, n=(m-2)⁄2 .

Proof : The above algorithm 2.4 construct the arithmetic progression of the form

S_2 = { m,m+a,m+2a,…,m+(n-2)a }with the path graph P_n such that n=(m-2)⁄2

Let f: VUE→ {1, 2, 3,…, 2n-1} be the bijective function defined as in the step 2

Then for all 1 ≤i≤ n-1;

$f(v_i) + f(v_{i+1})+f(v_iv_{i+1}) = 2i-1+2(i+1)-1+2n-2i = 2n+2i$

$= 2(m-2)/2 +2i = m-2+2i$

Now using values of i we get edge counts as {m, m+2, m+4,…, m+2n-4}

Which concludes that there exist an edge antimagic total labeling which is path graph for each arithmetic progression of the form S_2(jump 2) .

Example 4*:* The edge coont of P_6 {10, 13, 16, 19, 22} from an arithmetic progression { m,m+a,m+2a,…,m+(n-2)a } such that m≥7,a=3 , n=m-4, start from 10 with jump 3 is given in fig 4.

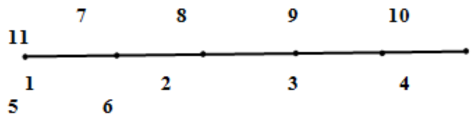

Fig:4

Algorithm- 2.7

Input: the arithmetic progression of the form S_3 = { m,m+a,m+2a,…,m+(n-2)a } such that m≥7,a=3.

Output: construction and edge antimagic total labeling for this arithmetic progression as a path graph.

Begin

Step 1: The vertex set V = {v_i: 1≤ i≤ n} and the edge set

E = {v_iv_{i+1} : 1 ≤i ≤ n-1 }

Step 2: f (v_i) = i , 1 ≤i ≤ n

Step 3: f (v_iv_{i+1}) = n+i 1 ≤i≤ n-1

End

Theorem 2.8 : The path P_n admit edge antimagic total labeling as the form of an arithmetic progression S_3 ={ m,m+a,m+2a,…,m+(n-2)a} when m≥7,a=3, n=m-4.

Proof : The above algorithm 2.7 construct the arithmetic progression of the form

S_3 ={ m,m+a,m+2a,…,m+(n-2)a } with the path graph P_n such that n=m-4

Let f:VUE→{1, 2, 3,…, 2n-1} be the bijective function defined as in the step 2 and 3

Then for all 1 ≤i≤ n-1;

$f(v_i) + f(v_{i+1})+f(v_iv_{i+1}) = i+i+1+n+i = n+3i+1$

$= m-4+3i+1 = m-3-3i.$

Now using values of i we get edge counts as {m, m+3, m+6,…, m+3n-6} Which concludes that there exist an edge antimagic total labeling which is path graph for each arithmetic progression of the form S_3 (jump 3).

Example 5: Edge antimagic total labeling of the path P_6 . The edge counts as S_4= {10, 14, 18, 22, 26}from arithmetic progression of the form S_4 = { m,m+a,m+2a,…,m+(n-2)a } such that m≥8, m≡0(mod 2) ,a=4, n=m-4 , start from 10 with jump 4 is given in fig 5.

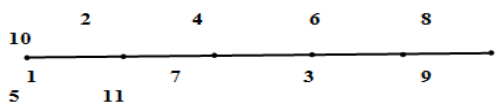

Fig 5

Algorithm- 2.10

Input: The arithmetic progression of the form

S_4 = { m,m+a,m+2a,…,m+(n-2)a } such that m≥8 & m≡0(mod2) ,a=4.

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Output: Construction and edge antimagic total labeling for this arithmetic progression as a path graph.
Begin

Step 1: The vertex set V = { v_i : $1 \le i \le n$} and the edge set E = {$v_i v_{i+1}$: $1 \le i \le n-1$ }

Step 2: $f(v_i) = \begin{cases} \text{if i is odd number go to step3} \\ \text{if i is even number go to step4} \end{cases}$

Step 3: $f(v_i) = i$ $\quad 1 \le i \le n$
Step 4: $f(v_i) = n+i-1$ $\quad 1 \le i \le n$
Step 5: $f(v_i v_{i+1}) = 2i$ $\quad 1 \le i \le n-1$
End

Theorem 2.11 *:*The path P_n admit edge antimagic total labeling as the form of an arithmetic progression S_4 ={ m,m+a,m+2a,…,m+(n-2)a} when m≥8,a=4, n=m-4 .

Proof : The above algorithm 2.10 construct the arithmetic progression of the form
S_4 = {m,m+a,m+2a,…,m+(n-2)a } with the path graph P_n such that n=m-4.

Let f: VUE→{ 1,2,3,…,2n-1} be the bijective function defined as in the step 2 and 3.

Then for all $1 \le i \le n-1$
$f(v_i) + f(v_{i+1}) + f(v_i v_{i+1}) = i+n+i+1-1+2i = n+4i$ if i is odd.
$f(v_i) + f(v_{i+1}) + f(v_i v_{i+1}) = n+i-1+i+1+2i = n+4i = m-4+4i$ if i is even. Now using values of i we get edge counts as {m, m+4, m+8,…, m+4n-8}.

Which concludes that there exist an edge antimagic total labeling which is path graph for each arithmetic progression of the form S_4(jump 4) .

III. ARITHMETIC PROGRESSION ON STAR GRAPH

Example 6: The edge counts of star graph $K_{1,11.}$ is given as S_4= {6, 10, 14, 18, 22,26,30,34,38,42,46} in the from of an arithmetic progression {m,m+a,m+2a,…,m+(n-1)a} such that m=6,a=4, start from 6 with jump 4 is given in fig 6.

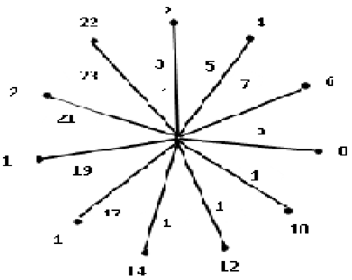

Fig 6: Edge antimagic total labeling of the star $K_{1,11.}$

Algorithm- 3.2
Input: The arithmetic progression of the form
S_4 ={m,m+a,m+2a,…,m+(n-1)a} such that m=6,a=4 .
Output: construction and edge antimagic total labeling for this arithmetic progression as a star graph.
Begin

Step 1: The vertex set V = { v_i : $1 \le i \le n+1$} and the edge set E = {$v_1 v_i$: $2 \le i \le n+1$ }
Step 2: $f(v_1) = 1$, $f(v_i) = 2(i-1)$;$2 \le i \le n+1$
Step 3: $f(v_1 v_i) = 2i-1$;$2 \le i \le n+1$
End

Theorem 3.3 : The star $K_{1,n}$ admit edge antimagic total labeling as the form of an arithmetic progression S_4 ={ m,m+a,m+2a,…,m+(n-1)a} when m=6,a=4.

Proof : The above algorithm 3.2 construct the arithmetic progression of the form S_4= { m,m+a,m+2a,…,m+(n-1)a } with the star graph $K_{1,n}$. Let f:VUE→ {1, 2, 3,…, n+1} be the bijective function defined as in the step 2 and 3. Then for all $2 \le i \le n+1$; $f(v_1) + f(v_i) + f(v_1 v_i) = 1+2(i-1)+2i-1 = 4i-2$.

Now using values of i we get edge counts as {6, 10, 14,…, 4n+2}that is *{ m,m+a,m+2a,…,m+(n-1)a}*.This concludes that there exist an edge antimagic total labeling which is star graph for each arithmetic progression of the form S_4 (jump 4).

IV. CONCLUSION

In above examples we show that it is possible to start with any number and directly one can find a maximum path for a sequence of numbers. A special cases on example 1.1 and 3.1 if we start from 6 and jump 6 we can get continuous path with any number we needed , if we start from 6 with jump 4 we can get continuous star with any number we needed. Our future work is to study arithmetic progression for other standard graph structures. Open Problem: Given an arithmetic progression, staring above 6 will there exist a labeled graph whose edge count generate it.

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