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Some New Classes of 3-Total Product Cordial Graphs

V. Sharon Philomena, A. Hemalatha
P. G. Department of Mathematics, Women's Christian College, Chennai
Email : sharon_philomena@yahoo.com

Abstract - In [3], Ponraj. R et al have defined the 3- Total Product Cordial of a graph G (V, E) as follows, Let f be a function from V(G) to{0, 1, ... k - 1}where k is an integer, $2 \leq k \leq$ V(G) . For each edge uv assign the label f (u) f (v) (mod k). f is called a k - Total Product cordial labeling if f (i) − f (j) ≤ 1, i, j ∈ {0, 1, . .k - 1} where f(x) denotes the total number of vertices and edges labeled with x(x = 0, 1, 2, ..., k-1). We prove that the 3-Total Product cordial labeling is a behaviour of F_n.

Keywords- Binary labeling, cordial labeling, Product cordial, k-Total product cordial .

I. INTRODUCTION

Graph labeling methods trace their origin to the graceful labeling introduced by Rosa [5] in 1967. In [4] Ponraj. R *et al* introduced a new graph labeling method called the 3-Total product cordial labeling, using the concept of k-Total product cordial labeling.

A. Definition

The vertex labeling f^* is said to be a binary labeling if f^* : V(G) → {0, 1} such that each edge xy is assigned the label $|f^*(x) – f^*(y)|$.

B. Definition

A binary vertex labeling of a graph *G* is called a cordial labeling if $| v_f (0) − v_f (1) | \leq 1$ and $| e_f (0) − e_f (1) | \leq 1$. A graph *G* is called cordial if it admits cordial labeling.

C. Definition

A binary vertex labeling of graph *G* with induced edge labeling f* : *E*(G) → {0, 1} defined by f*(e = uv) = f (u)f(v) is called a Product cordial labeling if $|v_f (0) – v_f (1)| \leq 1$ and $| e_f (0) – e_f (1) | \leq 1$. A graph is called Product cordial if it admits Product cordial labeling.

D. Definition

A total product cordial labeling of a graph G is a function f : (V(G) ∪ E(G)) → {0, 1} such that f(xy) = f (x) f(y) where x, y ∈ V(G), xy ∈ E(G) and the total number of 0's and 1's are balanced i.e. if v_f (i) and e_f (i) denote the set of vertices and edges which are labeled as i for i = 0, 1 respectively, then $|(v_f (0) + e_f (0)) − (v_f (1) + e_f (1))| \leq 1$. If there exists a total product cordial labeling of a graph G then it is called a Total product cordial graph.

E. Definition

Let f be a function from V(G) to {0, 1, …k-1 } where k is an integer, $2 \leq k \leq | V(G) |$. For each edge uv assign the label f (u)f(v)(mod k). f is called a k-Total product cordial labeling if $|f (i) - f (j)| \leq 1$, i, j ∈{0, 1, ...k-1 } where f (x) denotes the total number of vertices and edges labeled with x (x = 0, 1, 2... , k-1). A graph that admits a k-Total product cordial labeling is called k -Total product cordial graph. M. Sundaram *et al*(2012) [3] proved the following graphs are total product cordial. Every product cordial graph of even order or odd order and even size; all cycles except C_n. C_n with m edges appended at each vertex; The fan graph f_{n-1} where ($f_{n-1} = P_{n-1} + K_1$) and wheel graph W_n where ($W_n = C_{n-1} + K_1$). The Helm graph is H_n (obtained from a wheel graph by attaching a pendent edge at each vertex of the n-cycle).A wheel graph W_n with n vertices is defined to be the join of $C_{n-1} + K_1$ of a isolated vertex with the cycle of length n. The helm graph H_p is the graph obtained from a wheel graph by attaching a pendant edge at each vertex of the n-cycle. The flower graph F_n is the graph obtained from the helm by attaching each pendent edge vertex to the centre vertex of the Wheel (W_n). In this paper, we compute the 3-Total product cordial labeling on flower graphs.

II. MAIN RESULT

In this section we prove that the flower graph F_n is 3-Total product cordial graphs. We prove this in the following theorems.Theorem : The flower graph F_n is 3 - Total product cordial labeling.

Proof: Denote the central vertex of the flower graph, F_n as u. The vertex u is called the hub vertex of the flower graph. Denote the vertices in the cycle of the flower as $u_1, u_2, u_3, \ldots, u_{n-1}, u_n$ in the clockwise direction. Denote the end-vertices of the flower as $v_1, v_2, v_3, \ldots, v_{n-1}, v_n$ in the clockwise direction. Denote the edges incident with u as $c_1, c_2, c_3, \ldots, c_{m-1}, c_m$ in the clockwise direction. Denote the edges of the cycle in the flower as $e_1, e_2, e_3, \ldots, e_{m-1}, e_m$ in the clockwise direction. Denote the pendant edges of the flower as $d_1, d_2, d_3, \ldots, d_{m-1}, d_m$ in the clockwise direction. Denote the edge which is attached to the center vertex and with the pendent vertex as $a_1, a_2, a_3, \ldots, a_{m-1}, a_m$ again in the clockwise direction.

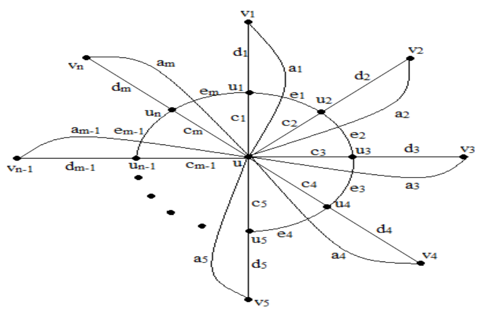

Figure 1: 3-Total product cordial of flower F_n

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In order to compute the 3-Total product cordial for the flower graph, we first label the vertex of the flower F_n and verify that these vertex labels are 3-Total product cordial as follows.
Let f be a function from V(G) to {0, 1, 2 }. For edge uv assign the label f(u)f(v) (mod3). Now we calculate the 3-Total product cordial for the flower graph as follows.

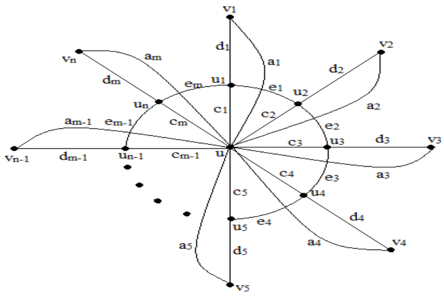

Figure:2

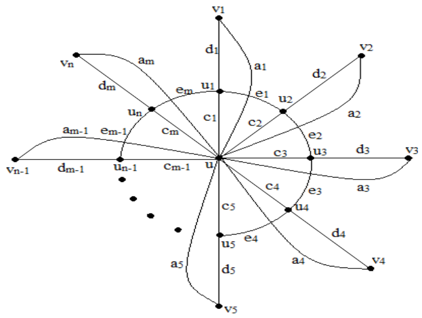

Figure:3

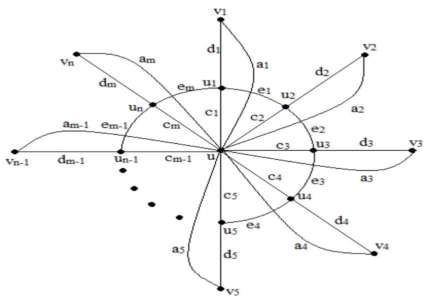

Figure:4

f is called a 3-Total product cordial labeling of G if $|f(i) - f(j)| \leq 1$, i, j ∈ {0, 1, 2 } where f(x) denotes the total number of vertices and the total number of edges labeled with x(x = 0, 1, 2). There are three cases for labeling the vertices of the flower in order to obtain the 3-Total product cordial for flower graph F_n

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A. Case 1

When $n \equiv 0 (\bmod 3)$

In this case we take n = 3t; t = 1,2, 3, ….

Define the vertex labels of flower F_n as follows.

f(u) = 0

$$f(u_i) = 2 ; 1 \leq i \leq n$$
$$f(v_i) = 2 ; 1 \leq i \leq n \quad (1)$$

Case 1: When $n \equiv 0 (\bmod 3)$, If t = 3 ; n = 9 Here f (0) = 6t + 1 =19, f (1) = f (2) = 6t = 18

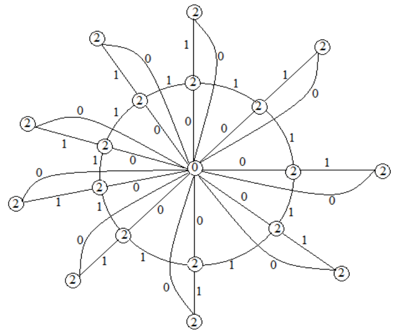

Figure:5 Total product cordial on Flower graph F_9

From the equation (1) the total number of vertices and edges labeled 0 is calculated as

$$f(0) = 6t + 1 \quad (2)$$

From the equation (1) the total number of vertices and edges labeled 1and 2 is calculated as

$$f(1) = f(2) = 6t \quad (3)$$

Therefore f is 3–Total product cordial labeling.

B. Case 2

When $n \equiv 1(\bmod 3)$

In this case we take n-1 = 3t; t = 1,2, 3, ….

Define the vertex labels of flower F_n as follows.

f(u) = 0

$$f(u_i) = 2 ; 1 \leq i \leq n$$
$$f(v_i) = 2 ; 1 \leq i \leq n \quad (4)$$

From the equation (4) the total number of vertices and edges labeled 0 is calculated as

$$f(0) = 6t + 3 \quad (5)$$

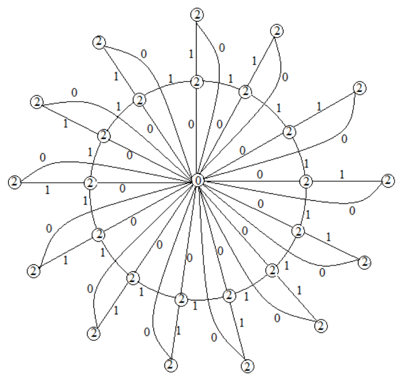

Figure:6 Total product cordial on Flower graph F_{13}

Case 2 : When $n \equiv 1(\bmod 3)$, If t = 4 ; n = 13, Here f (0) = 6t + 3 = 27, f (1) = f (2) = 6t + 2 = 26

From the equation (5) the total number of vertices and edges labeled 1 and 2 is calculated as

$$f(1) = f(2) = 6t + 2 \quad (6)$$

Therefore f is 3-Total product cordial labeling

C. Case 3

When $n \equiv 2(\bmod 3)$

In this case we take n-2 = 3t; t = 1,2, 3, ….

Define the vertex labels of flower F_n as follows:

f(u) = 0

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$f(u_i) = 2 \; ; \; 1 \leq i \leq n$

$f(v_i) = 2 \; ; \; 1 \leq i \leq n$ (7)

From the equation (7) the total number of vertices and edges labeled 0 is calculated as

$f(0) = 6t + 5$ (8)

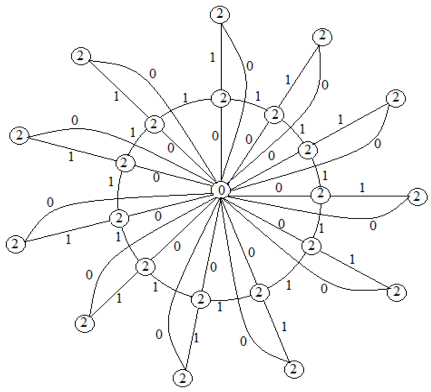

Figure:7 Total product cordial on Flower graph F_{11}

From the equation (3.8) the total number of vertices and edges labeled 1 and 2 is calculated as

$f(1) = f(2) = 6t + 4$ (9)

Hence the graph F_n is 3-Total product cordial labeling.

The illustrations for the various cases of the above theorem are given in the appendix.Further the sub-division of flower graph which admits 3 – total product cordial is been proved and Cartesian product of flower graph with a complete graph K_1 can be computed .

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