

# Double Diffusive Convection in a Rectangular Enclosure in the Presence of Heat Source Double-Diffusive Natural Convective Flow in a Rectangular Enclosure in the Presence of Heat Source

K.Valarmady<sup>1</sup>, S.Subbulakshmi<sup>2</sup>, R.Vasanthakumari<sup>3</sup>, K.Thirumurugan<sup>4</sup>

<sup>1</sup>Assistant Professor, Department of Mathematics, Tagore Arts College

<sup>2</sup>Assistant Professor, Department of Mathematics, D.G.G.A College(W),Mayiladuthurai

<sup>3</sup>Associate Professor, Department of Mathematics, K.M.C.P.G.S, Pudukcherry

<sup>4</sup>Research Scholar, Bharathiar University, Coimbatore

Abstract - Double-diffusive natural convective flow in a rectangular enclosure with the shortest sides being insulated and impermeable is investigated numerically. Constant temperatures and concentration are imposed along the longest sides of the enclosure. Laminar regime is considered under steady-state condition. The transport equations for continuity, momentum, energy are solved using the finite volume technique. The numerical results are reported for the effect of thermal Rayleigh number on the contours of streamline, temperature, and concentration. In addition, results for the average Nusselt and Sherwood numbers are presented and discussed for various parametric conditions. This study is done for constant Prandtl number,  $Pr = 0.7$ ; aspect ratio,  $A = 2$  and Lewis number,  $Le = 2$ . Computations are carried out for thermal Rayleigh number ranging from  $10^3$  ranging from  $-40$  F  $40$ , buoyancy ratio ranging from  $-5$  N  $5$  and the Hartmann number ranging from  $0$  Ha  $70$  to  $5 \times 10^5$ , dimensionless heat generation and absorption coefficients

Key Words - Double Diffusive convection; rectangular enclosure; Internal heat generation; aspect

## I. INTRODUCTION

Natural Convection induced by internal heat generation has wide applications in various fields such as geo-physics and energy related engineering problems. Acharya and Goldstein (1) studied numerically the inner heat generation with inclined cavity Rahman and Sharif (2) studied the same with heated bottom and cooled top surfaces and insulated sides. Oztop and Bilgen (3) studied numerically heat transfer in a differentially heated, partitioned, square cavity. Double diffusive convection plays a major role in various fields like oceanography, astrophysics, drying process and crystal growth process. These are found in the publications of Ostrach (4), Costa (5) and Nishimura (6).

Nithyadevi and Yang (7) studied numerically the effect of double diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients around the density maximum. The present work deals with double diffusive convection in a rectangular enclosure in the presence

of heat source. This may occur in such related to nuclear reactor cores, fire and combustion modeling, electronic chips and semiconductor wafers.

## II. MATHEMATICAL MODEL

A two dimensional rectangular enclosure of height  $H$  and width  $L$  is filled with a binary mixture of Gas. The longest sides are maintained at constant and uniform different levels of temperatures and concentrations giving raise to double diffusive free convection flow field. The top and bottom surfaces are assumed to be adiabatic and impermeable. The fluid is assumed to be Newtonian, incompressible, heat generating or absorbing and viscous. Boussinesque approximation leads to

$$\rho = \rho_0 [1 - \beta_T(T - T_c) - \beta_s(c - c_1)] \tag{1}$$

The governing equations of the problem under consideration are based on the balance laws of mass, linear momentum, concentration and thermal energy in two dimensions steady state. In the light of assumptions mentioned above, the continuity, momentum, energy and concentration in two-dimensional equations can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] + g\beta_T(T - T_c) - \beta_s(c - c_1) \tag{3}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + g\beta_T(T - T_c) - \beta_s(c - c_1) - \frac{\sigma \beta^2}{\rho} \nu \tag{4}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{Q_0}{\rho c_p} (T - T_c) \tag{5}$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \left[ \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right] \tag{6}$$

The boundary conditions for the problem could be written as

- At  $x = 0, u = v = 0.0, T = T_h$  and  $c = c_h$
- At  $x = L, u = v = 0.0, T = T_c$  and  $c = c_l$
- And at  $y = 0$  and  $y = H$
- 

$$u = v = \frac{\partial T}{\partial y} = \frac{\partial c}{\partial y} = 0 \tag{7}$$

The boundary conditions and the governing equations are non dimensionless variables

$$X^* = \frac{x}{L}, Y^* = \frac{y}{L}, U = \frac{uL}{\alpha}, v = \frac{vL}{\alpha}, P = \frac{pL^2}{\rho^* \alpha^2}, \theta = \frac{(T-T_c)}{(T_h-T_c)} \text{ and } C = \frac{(c-c_l)}{(c_h-c_l)} \tag{8}$$

After employing dimensionless variables mentioned above, the resulting dimensionless governing equations can be written as

$$\frac{\partial U}{\partial X^*} + \frac{\partial V}{\partial Y^*} = 0 \tag{9}$$

$$U \frac{\partial U}{\partial X^*} + V \frac{\partial V}{\partial Y^*} = -\frac{\partial P}{\partial Y^*} + Pr \left[ \frac{\partial^2 U}{\partial X^{*2}} + \frac{\partial^2 U}{\partial Y^{*2}} \right] + [RaPr(\theta - NC)] \tag{10}$$

$$U \frac{\partial U}{\partial X^*} + V \frac{\partial V}{\partial Y^*} = -\frac{\partial P}{\partial Y^*} + Pr \left[ \frac{\partial^2 V}{\partial X^{*2}} + \frac{\partial^2 V}{\partial Y^{*2}} \right] + [RaPr(\theta - NC)] + Ha^2 Pr \times V \tag{11}$$

$$U \frac{\partial \theta}{\partial X^*} + V \frac{\partial \theta}{\partial Y^*} = \left[ \frac{\partial^2 \theta}{\partial X^{*2}} + \frac{\partial^2 \theta}{\partial Y^{*2}} \right] + \phi \times \theta \tag{12}$$

$$\left[ \frac{\partial^2 C}{\partial X^{*2}} + \frac{\partial^2 C}{\partial Y^{*2}} \right] = \frac{1}{Le} \left[ \frac{\partial^2 C}{\partial X^{*2}} + \frac{\partial^2 C}{\partial Y^{*2}} \right] \tag{13}$$

Where Pr is the Prandtl number. Ra is the thermal Rayleigh number. N is the buoyancy ratio  $= \beta_s[(C_h - C_l)/\beta_T[(T_h - T_c)]$ ,  $\phi$  is the dimensionless heat generation or absorption coefficient  $= (Q_0 L^2)/(\rho C_p \alpha)$ , and Le is the Lewis number,  $Le = \alpha/D$ .

- At  $X^* = 0, u = v = 0.0, \theta = 1$  and  $c = 1$
- At  $X^* = 1, u = v = 0.0, \theta = c = 0$
- And at  $Y^* = 0$  and  $Y^* = Aspect Ratio$
- 

$$U = V = \frac{\partial \theta}{\partial Y^*} = \frac{\partial c}{\partial Y^*} = 0 \tag{14}$$

The Nusselt and Sherwood numbers calculated as average values and evaluated along hot isothermal wall of cavity are given by

$$Nu = -\frac{1}{A} \int_0^A \left( \frac{\partial \theta}{\partial X^*} \right)_{X^*=0} dY^* \tag{15}$$

$$Sh = -\frac{1}{A} \int_0^A \left( \frac{\partial C}{\partial X^*} \right)_{X^*=0} dY^* \tag{16}$$

A. The effect of buoyancy ratio on the local Nusselt and Sherwood Numbers

It is to explore the effect of buoyancy ratio on the distribution for both local Nusselt and Sherwood numbers over the hot wall of the cavity. The following parameters are kept constant  $Pr=0.7, Le=2, N=1, \phi = 0, Ra=10^5$  and  $-5 \leq N \leq 5$  for the buoyancy ratio range of  $0 \leq N \leq 5$  the local Nusselt has maximum values at the cavity bottom and its value decreases as we move upwards. The absolute value of the temperature gradient (for the above buoyancy ratio range) has maximum values at the cavity bottom and decreases as we move upwards reaching the minimum value at the cavity top. On the other hand, for the case of  $N=-1$ , the local Nusselt number shows no change and it is kept constant all the value along the hot wall. This is an indication for pure conduction condition.

In addition for the buoyancy ratio range of  $-3 \leq N \leq -5$ , opposite contribution for the previous buoyancy ratio range occurs where the local Nusselt number has minimum value at the cavity bottom and it starts to increase as we move upwards. The local Nusselt number has maximum values at the cavity top. Further more, it shows the similar contributions for the effect of buoyancy ratio on the local Sherwood number. The main difference is that the local Sherwood number generally has higher values than the local Nusselt number.

B. The effect of buoyancy ratio on the average Nusselt and Sherwood numbers

It plots the effect of buoyancy on the average Nusselt and Sherwood numbers. That there is a critical buoyancy ratio  $N_{cr}$  where the values of average Nu and Sh numbers are minimum. Both of the average Nusselt and Sherwood number are likely to decrease with increasing values of N for  $N > N_{cr}$ .

C. The effect of heat generation or absorption coefficients ( $\phi$ )

Process may often develop inside the enclosures or the cavities studied that result in liberation or absorption of heat. Processes of this kind are exemplified by liberation of joulean heat by an electric current flowing through conductor, volumetric liberation of heat as a result of heat result of heat liberating element of atomic reactors, liberation or absorption of heat during many chemical reactions and reactions including phase changes. Possible heat generation effects may change the temperature distribution and, therefore the particle deposition rate. This may occur in such related to nuclear reactor cores, fire and combustion modeling, electronic chips and semiconductor wafers.

This part discuss the effect of heat generation or absorption coefficient on double-diffusive natural convection in an inclined rectangular enclosure in the presence of magnetic field and heat source. In this section the Prandtl number; Pr is kept constant at  $Pr=0.7$ , Lewis number;  $Le=2$ , the Hartmann number;  $Ha=50$  and the thermal Rayleigh number;  $Ra=10^5$  Computations are carried out for heat generation or absorption coefficient ( $\phi$ ) range of  $-40 \leq \phi \leq 40$  and buoyancy ratio

range of  $-5 \leq N \leq 5$ . The numerical results for the streamline, isotherm and isoconcentration contours for various values of heat generation or absorption coefficient will be presented and discussed. In addition, the effect of the heat generation or absorption coefficient on the average and local Nusselt and Sherwood numbers is also discussed at various conditions.

#### D. The Effect of heat generation and absorption coefficient ( $\phi$ ) on the local Nusselt and Sherwood Numbers

It is explore the effect of heat generation or absorption phenomena on the distribution for both local Nusselt and Sherwood numbers over the hot wall. On the whole, the local Nusselt number has maximum values at the cavity bottom and its value decreases upwards. For the same position on the hot wall the local Nusselt number decreases as ( $\phi$ ) increases. The value of the local Nusselt number is positive for long as heat is absorbed from the cavity and also when there is no heat generation or absorption or applied to the cavity ( $\phi = 0$ ). Nusselt number has maximum value at the position. In the contrast, the heat source or sink has no significant effect on the local Sherwood number.

#### E. The effect of heat generation or absorption coefficient ( $\phi$ ) on the average Nusselt and Sherwood Numbers

It plots the effect of heat generation or absorption coefficient ( $\phi$ ) on the average Nusselt and Sherwood numbers. It has the heat generation or absorption coefficient ( $\phi$ ) increases, the average Nusselt number decreases and again the negative values are due to the reverse action of the heat generation as the heat transfer is shifted instead of being form the hot wall to the fluid to the hot wall. In addition both the heat generation and heat absorption coefficients faintly decrease the average Sherwood number.

### III. NUMERICAL SOLUTION

Using finite volume method the total differential equations governing double diffusive convection are reduced to a system of simultaneous algebraic equations. Steady state solutions are obtained using the under-relation techniques. The differentiation equations were solved by Gauss-Seidel method.

### IV. CONCLUSION

The following are the main conclusions of the present work: For lower values of the thermal Rayleigh number the conduction regime was dominant. Increasing the source term in the momentum equation, by increasing the thermal Rayleigh number, always led to increases on the heat and mass transfer performance of the enclosure. The effect of thermal Rayleigh number on the horizontal cavity was noticed by the increased in the number of convection roles developed within the cavity. At buoyancy ratio equal to -1 the thermal and compositional buoyance effects are equal to and in opposite directions which in return makes the cavity encounter low flow circulation and subsequently very lower rates of heat and mass transfer. The critical value of the bouyacy ration is equal to -1. The value of the average Nusselt and Sherwood numbers tends to increase

with increasing the absolute values of buoyancy ratio. The magnetic field reduces the heat transfer and fluid circulation within the enclosure due to retardation effect of the electromagnetic body force. The retardation effect of electromagnetic body force is noticeable from the decrease in the strength of the circulation cell in the flow field and from the gradual shifting of the isotherm and isoconcentration contours to the vertical distribution in the cavity core which in returns show and indication for a quasi-conduction regime approach. When a heat sources applied to the cavity the results showed that the heat transfer direction was reversed. This reverse action of the heat generation generally decreases the heat transfer within the cavity. The local and average Nusselt numbers have negative values when a heat source is applied to the cavity. The negative sign is an indication for the reverse action in the flow caused by the heat source thus the average Nusselt number decreases as the heat generation coefficient increases. The results showed that both the heat generation and heat absorption coefficients had no significant effect on the average Sherwood number.

### REFERENCES

- [1] S. Acharya, R.J. Goldstein, Natural convection in an externally heated square box containing internal energy sources, *J. Heat Transfer* 107 (1985) 855 - 866.
- [2] S. Ostrach, Natural convection with combined driving forces, *Phys. Chem. Hydrodyn.* 1 (1980) 233-247.
- [3] N. Nithyadevi, Ruy-Jen Yang, Double diffusive natural convection in a partially heated enclosure with Soret and Dufour effects, *J. Heat Fluid Flow* 30 (2009) 902-910.
- [4] M. Rahman, M.A.R. Sharif, Numerical study of laminar natural convection in inclined rectangular enclosures of various aspect ratios, *Numer. Heat Transfer A* 44 (2003) 355-373.
- [5] H. Oztop, E. Bilgen, Natural convection in differentially heated and partially divided square cavities with internal heat generation, *Int. J. Heat Fluid Flow* 27 (2006) 466-475.
- [6] T. Nishimura, M. Wakamatsu, A.M. Morega, Oscillatory double diffusive convection in a rectangular enclosure with combined horizontal temperature and concentration gradients, *Int.J. Heat Mass Transfer* 41(1998) 1601 - 1611.
- [7] N. Nithyadevi, Ruy-jen Yang, Double diffusive nature convection in a partially heated enclosure with Soret and Dufour Effects, *Int.J. Heat Fluid flow* 30(2009) 902 -910