Exact Double Domination in Graph

L.JethruthEmelda Mary¹, S.Kalaiselvi²
¹,²PG and Research Department of Mathematics
St.Joseph’s Arts & Science College(Autonomous),Cuddalore,India.
E-mail:jethruth15@gmail.com, s.k.selvi01@gmail.com

Abstract- In this paper, we deals about exact double domination in graphs. In a graph a vertex is said to dominate itself and all its neighbours. A double dominating set is exact if every vertex of G is dominated exactly twice. If a double dominating set exist then all such sets have the same size and bounds on this size. We established a necessary and sufficient condition of exact double dominating set in a connected cubic graph with application.

Keywords - Double domination, exact double domination, connected cubic graph.

I. INTRODUCTION

In a graph G=(V,E),a subset S ⊆ V is a dominating set of G if every vertex v of V−S has a neighbour in S. The domination number γ(G) is the minimum cardinality of a dominating set of G. Harary and Haynes defined and studied the concept of double domination, which generalizes domination in graphs. In a graph G = (V,E), a subset S of V is a doubly dominating set of G if every vertex v ∈ V,either v is in S and has at least one neighbour in S or v is in V−S and has at least two neighbours in S. The double domination number γ²(G) is the minimum cardinality of a doubly dominating set of G. Double domination was also studied in [2,3,4].

Harary and Haynes defined an sufficient condition for doubly dominating set as a subset S of V such that each vertex of V is dominated by exactly two vertices of S. We will prefer here to use the phrase exact doubly dominating set. Every graph G = (V,E) with no isolated vertex has a doubly dominating set then all such sets have the same size and we give some bound on this number. Finally, we construct the characterization of those trees that admit an exact doubly dominating set and we establish a necessary and sufficient condition for the existence of an exact doubly dominating set in a connected cubic graph. We denote respectively by n, δ and ∆ the order (number of vertices), minimum degree and maximum degree of a graph G.

II. DEFINITIONS

A. Dominating set
A set S of vertices of G is a dominating set of G if every vertex of G is dominated by atleast one vertex of S.

B. Domination number
The minimum cardinality among the dominating sets vertex of G is called the dominating number of G and is denoted by γ(G).

C. Double domination number
The double domination number γ²(G) is the minimum cardinality of a doubly domination set of G.

D. Exact double domination
In a graph G = (V,E),a subset S ⊆ V is a dominating set of G. If for vertex v∈ V either vis in S and has at least one neighbour in S or v is in V−S and has at least two neighbours in S is called exact double domination.

III. GRAPHS WITH EXACT DOUBLE DOMINATING SETS

Result: 1
γ²(cₙ)=[²n+1
2
]

Solution:
Let γ²-double domination number
n-number of vertices
[2n+1]
−upper integral part
Cₙ -cycle of order n
If n=3, then S₁ = {a,b}
Clearly,S₁ is a minimum doubly dominatin

γ²(c₃)=[2·3
2
]=2

For n = 4, we have S₂={a,c,d}

Clearly, S₂ is a minimum double dominating set of G

γ²(c₄)= [2·4
2
]=[3]=2

For n=5, we get S₅={a,b,e,d,}

S₅ is a minimum double dominating set of G
When \( n = 6 \), \( S_4 = \{a,b,c,e,f\} \) is a minimum double dominating set of \( G \).

Consider \( G = K_5 \) graph.

In a complete graph \( \gamma_2(G) = 2 \), and \( n = 5, s = 4 \)

\( n - S + 1 = 5 - 4 + 1 = 2 \)

\( \gamma_2(G) = n - S + 1 \)

Suppose \( G \) is a tree (ie) \( G \) is a acyclic connected graph.

For \( G \) is a Star graph then

\( \gamma_2(G) = n - S + 1 \)

n=7, S =1

ie) \( n - S + 1 = 7 \).

Result: 2

If \( G \) is a path then

\( \gamma_2(P_n) = n - S + 1 \)

\( \gamma_2(P_n) < n - S + 1 \)

Theorem: 3

Let \( G \) be a connected cubic graph, then \( G \) has an exact doubly dominating set if and only if \( G \) has a perfect matching \( M \) such that associated graph \( G_M \) is an equitable bipartite graph.

Proof :-

Let \( G \) be a connected cubic graph with an exact doubly dominating set \( S \).

Consider \( G = K_5 \) graph.

In a complete graph \( \gamma_2(G) = 2 \), and \( n = 5, s = 4 \)

\( n - S + 1 = 5 - 4 + 1 = 2 \)

\( \gamma_2(G) = n - S + 1 \)

Suppose \( G \) is a tree (ie) \( G \) is a acyclic connected graph.

For \( G \) is a Star graph then

\( \gamma_2(G) = n - S + 1 \)

n=7, S =1

ie) \( n - S + 1 = 7 \).

Result: 2

If \( G \) is a path then

\( \gamma_2(P_n) = n - S + 1 \)

\( \gamma_2(P_n) < n - S + 1 \)

Theorem: 3

Let \( G \) be a connected cubic graph, then \( G \) has an exact doubly dominating set if and only if \( G \) has a perfect matching \( M \) such that associated graph \( G_M \) is an equitable bipartite graph.

Proof :-

Let \( G \) be a connected cubic graph with an exact doubly dominating set \( S \).

Consider \( G = K_5 \) graph.

In a complete graph \( \gamma_2(G) = 2 \), and \( n = 5, s = 4 \)

\( n - S + 1 = 5 - 4 + 1 = 2 \)

\( \gamma_2(G) = n - S + 1 \)

Suppose \( G \) is a tree (ie) \( G \) is a acyclic connected graph.

For \( G \) is a Star graph then

\( \gamma_2(G) = n - S + 1 \)

n=7, S =1

ie) \( n - S + 1 = 7 \).

Result: 2

If \( G \) is a path then

\( \gamma_2(P_n) = n - S + 1 \)

\( \gamma_2(P_n) < n - S + 1 \)

Theorem: 3

Let \( G \) be a connected cubic graph, then \( G \) has an exact doubly dominating set if and only if \( G \) has a perfect matching \( M \) such that associated graph \( G_M \) is an equitable bipartite graph.

Proof :-

Let \( G \) be a connected cubic graph with an exact doubly dominating set \( S \).

Consider \( G = K_5 \) graph.

In a complete graph \( \gamma_2(G) = 2 \), and \( n = 5, s = 4 \)

\( n - S + 1 = 5 - 4 + 1 = 2 \)

\( \gamma_2(G) = n - S + 1 \)

Suppose \( G \) is a tree (ie) \( G \) is a acyclic connected graph.

For \( G \) is a Star graph then

\( \gamma_2(G) = n - S + 1 \)

n=7, S =1

ie) \( n - S + 1 = 7 \).

Result: 2

If \( G \) is a path then

\( \gamma_2(P_n) = n - S + 1 \)

\( \gamma_2(P_n) < n - S + 1 \)

Theorem: 3

Let \( G \) be a connected cubic graph, then \( G \) has an exact doubly dominating set if and only if \( G \) has a perfect matching \( M \) such that associated graph \( G_M \) is an equitable bipartite graph.

Proof :-

Let \( G \) be a connected cubic graph with an exact doubly dominating set \( S \).
Since every vertex of $V-S$ has exactly two neighbours in $S$, the subgraph induced by $V-S$ is 1-Regular and its forms a matching $M_2$.

Thus $G$ admits a perfect matching $M=M_1 \cup M_2$.

Each edge of $E-M$ joins a vertex of $S$ with a vertex of $V-S$, and the bipartite subgraph is 2-regular.

Thus the graph $G_M$ associated with $M$ is an equitable bipartite with equitable bipartition $(M_1, M_2)$.

Conversely, Let $M$ be a perfect matching of a connected cubic graph $G$ such that the associated graph $G_m$ is equitable bipartite, with equitable bipartition $(A, B)$. Let $A_m$ (resp. $B_m$) be the vertices of $G$ that are contained in the edges corresponding to the vertices of $A$ (resp. $B$).

Consequently, $A_m$ and $B_m$ are two disjoint applied of critical exact doubly dominating sets of $G$.

IV. APPLICATION OF DOUBLE DOMINATION
CRITICAL TREES AND CYCLES

Often when it is difficult to characterize graphs with particular parameters, it is helpful to restrict one’s attention to trees. It has been found that no tree is domination or total domination edge critical.

First we must define a star and double star. A star is a tree with exactly one vertex that is not a leaf.

Where $\gamma_2(K_{1,6}) = 7$ while $\gamma_2(K_{1,6}+e) = 6$ for any edge $e \in E(K_{1,6})$.

In general, for the star $K_{1,m}$, where $\gamma_2(K_{1,m}) = m + 1$, then $\gamma_2(K_{1,m} + e) = m + 1$ for any edge $e \in E(K_{1,m})$. A double star is a tree with exactly two vertices that are not leaves in figure 2. Using this information we characterize the double domination edge critical trees.

**Proposition : 1**
A tree $T$ is double domination edge critical if and only if $T$ is a star or a double star.

**Proposition : 2**
Let $G' = G + uv$ for any $uv \in E(G)$ and again the darkened vertices represent a double dominating set. Note that any edge added will decrease $\gamma_2(G)$ by one.

Let $G$ be a star graph of order 7.

$\gamma_2(G) = 7$

When an edge is added to the star graph, we have $\gamma_2(G+uv) = 6$.

V. CONCLUSION

We discussed double domination, exact double domination and connected cubic double domination of graph and we established the necessary and sufficient condition of the existence of connected cubic double domination graph of graph with different parameters and also we verified that $\gamma_2(G)$ will decrease by one when any edge is added.

REFERENCES