International Journal of Computing Algorithm
Volume: 03 Issue: 03 December 2014 Pages: 240-242
ISSN: 2278-2397

Exact Double Domination in Graph

L.JethruthEmelda Mary [1], S.Kalaiselvi[2]
[1,2]PG and Research Department of Mathematics
St.Joseph's Arts & Science College(Autonomous),Cuddalore,India.
E-mail:jethruth15@gmail.com, s.k.selvi01@gmail.com

Abstract- In this paper, we deals about exact double domination in graphs. In a graph a vertex is said to dominate itself and all its neighbours. A double dominating set is exact if every vertex of G is dominated exactly twice. If a double dominating set exist then all such sets have the same size and bounds on this size. We established a necessary and sufficient condition of exact double dominating set in a connected cubic graph with application.

Keywords - Double domination, exact double domination, connected cubic graph.

I. INTRODUCTION

In a graph G=(V,E),a subset S ⊆V is a dominating set of G if every vertex v of V−S has a neighbour in S.The domination number γ(G) is the minimum cardinality of a dominating set of G.Harary and Haynes defined and studied the concept of double domination, which generalizes domination in graphs.In a graph G =(V,E), a subset S of V is a doubly domination set of G if every vertex v ∈V,either v is in S and has atleast one neighbour in S or v is in V−S and has atleast two neighbours in S.The double domination number $\gamma_2(G)$ is the minimum cardinality of a doubly dominating set of G. Double domination was also studied in [2,3,4].

Harary and Haynes defined an sufficient condition for doubly dominating set as a subset S of V such that each vertex of V is dominated by exactly two vertices of S.We will prefer here to use the phrase exact doubly dominating set. Every graph G = (V,E) with no isolated vertex has a doubly dominating set,if a graph G admits an exact doubly dominating set then all such sets have the same size and we give some bound on this number.Finally, we construct the characterization of those trees that admit an exact doubly dominating set and we establish a necessary and sufficient condition for the existence of an exact doubly dominating set in a connected cubic graph.We denote respectively by n, δ and Δ the order (number of vertices), minimum degree and maximum degree of a graph G.

II. DEFINITIONS

A. Dominating set
A set S of vertices of G is a dominating set of G if every vertex of G is dominated by atleast one vertex of S.

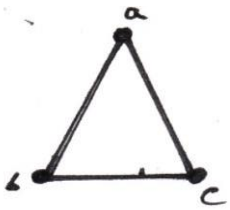
B. Domination number
The minimum cardinality among the dominating sets vertex of G is called the dominating number of G and is denoted by γ(G).

C. Double domination number
The double domination number $\gamma_2(G)$ is the minimum cardinality of a doubly domination set of G.

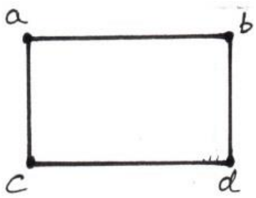
D. Exact double domination
In a graph G =(V,E),a subset S ⊆V is a double dominating set of G.If for vertex
v∈ V either vis in S and has atleast one neighbour in S (or) v is in V-S and has atleast two neighbours in S is called exact double domination.

III.GRAPHS WITH EXACT DOUBLE DOMINATING SETS

Result:1
$\gamma_2(c_n)=\left\lceil \frac{2n}{3} \right\rceil$
Solution:
Let γ_2-double domination number
n-number of vertices
$\lceil\ \rceil$ −upper integral part
C_n -cycle of order n
If n=3, then S_1 = {a,b}
Clearly,S_1 is a minimum doubly dominating set of G.
$\gamma_2(c_3)=\left\lceil \frac{2\times 3}{3} \right\rceil=2$



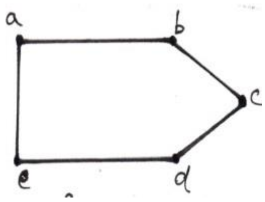
For n = 4,we have S_2={a,c,d}



Clearly, S_2 is a minimum double dominating set of G

$\gamma_2(c_4) = \left\lceil \frac{2\times 4}{3} \right\rceil=\left\lceil \frac{8}{3} \right\rceil =\lceil 2.6\rceil = 3$

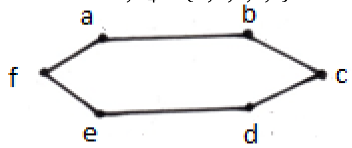
For n=5,we get S_3={a,b,e,d,}



S_3 is a minimum double dominating set of G

$\gamma_2(p_5) = 2\left\lceil\frac{5}{3}\right\rceil = 2\lceil 1.6\rceil = 2\times 2 = 4$

When n = 6 ,S_4 = {a,b,c,e,f}



S_4 is a minimum double dominating set of G

$\gamma_2(c_6) = \left\lceil\frac{2\times 6}{3}\right\rceil = \lceil 4\rceil = 4$

In General, $\gamma_2(c_n) = \left\lceil\frac{2\times n}{3}\right\rceil$

3.1 Exact doubly domination sets:

Theorem:1

If G has an exact doubly dominating set then all such sets have the same size.

Proof: Let D_1,D_2 be two exact doubly dominating sets of G. Let I =$D_1 \cap D_2$, and let X_0 and X_1 be the subsets of D_1−I such that every vertex of X_0 has zero neighbours in I and every vertex of X_1 has one neighbour in I.

Clearly $D_1 - I = X_0 \cup X_1$.

We define a subsets Y_0 and Y_1 of D_2 −I.

We claim that $|X_1| = |Y_1|$.

let x be any vertex of X_1,adjacent to a vertex z∈I.Since D_2 is an exact doubly dominating set, z has a unique neighbour y in D_2.We have y ∈ D_2 −I, for otherwise z has two neighbours x,y in D_2,which is a contradiction.Thus y∈Y_1.The symmetric argument holds for every vertex of Y_1, and so $|X_1| = |Y_1|$.Since D_2 is an exact doubly dominating set, every vertex of X_1 has exactly one neighbour in $Y_0 \cup Y_1$ and every vertex of X_0 has exactly two neighbours in $Y_0 \cup Y_1$.The vertices of Y_1 and Y_0. This implies

$|X_0| = |Y_0|$, thus $|D_1| = |D_2|$.

The size of an exact doubly dominating set with the order and minimum degree δ of a graph G.

Theorem: 2

Every graph G without isolated vertices satisfies,

γ_2 (G) = n – S +1

where

n– Number of vertices

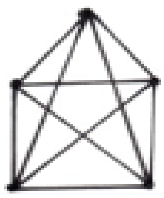
S – Minimum degree

γ_2 – double domination number

Example:1

If G is complete, then

γ_2 (G) = n-S+1



Consider G = K_5 graph,

In a complete graph γ_2(G)=2,and n=5,s=4

n-S+1=5-4+1=2

γ_2(G) = n-S+1

Suppose G is a tree

(ie) G is a acylic connected graph

For G is a Star graph then

γ_2(G) = n– S +1

n=7, S =1

ie) n-S+1=7.

Result:2

If G is a path then

$$\gamma_2(P_n) = \begin{cases} 2\left\lceil\frac{n}{3}\right\rceil + 1 \text{ if } n \equiv (mod 3) \\ 2\left\lceil\frac{n}{3}\right\rceil \text{ otherwise} \end{cases}$$

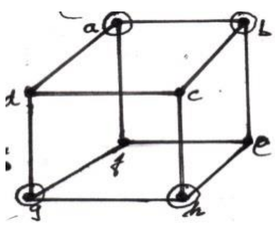
and hence $\gamma_2(P_n)$ < n-S+1

Theorm:3

Let G be a connected cubic graph, then G has an exact doubly dominating set if and only if G has a perfect matching M such that associated graph G_M is an equitable bipartite graph.

Proof :-

Let G be a connected cubic graph with an exact doubly dominating set S.

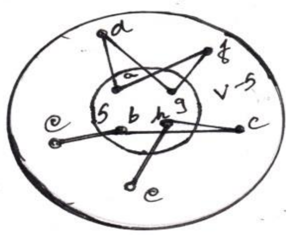


S={a,b,g,h}

So s induces a 1-regular graph

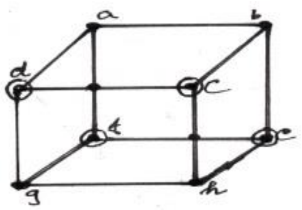
1-regular means every vertex has degree1. whose edge form a matching M_1

and every vertex of S has two neighbours in V-S

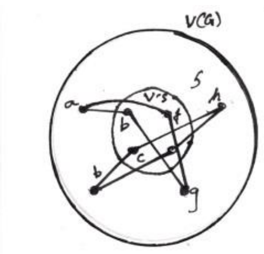


M_1

Since every vertex of V-S has exactly two neighbours in S,thesubgraph induced by V-S is 1-Regular and its forms a matching M_2.



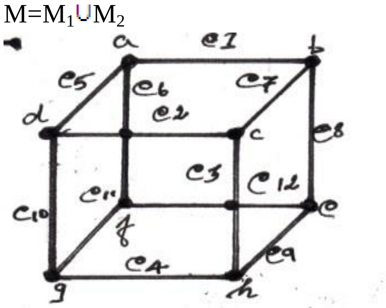
v-s={d,c,f,e}



M_2

Thus G admits a perfect matching

$M=M_1 \cup M_2$



$M_1=\{e_1=ab,\ e_4=gh\}$

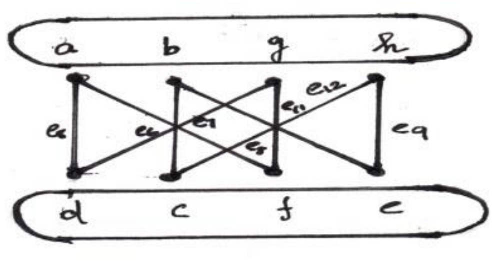
$M_2=\{e_2=dc, e_3=fe\}$

Each edge of E-M joins a vertex of S with a vertex of V-S, and the bipartite subgraph is 2- regular.

[$M=\{e_1, e_4, e_2, e_3\}$

$E-M = \{e_1, e_2, \ldots\ldots e_{12}\} - \{e_1, e_4, e_2, e_3\}$

$=\{e_5, e_6, e_7 \ldots\ldots\ldots\ldots e_{12}\}$]



$G_M= (S, V\text{-}S; E\text{-}M)$

So, $|S| = |V\text{-}S|$, and so $|M_1|=|M_2|$

Thus, the graph G_M associated with M is an equitable bipartite with equitable bipartition (M_1, M_2).

Conversely,

Let M be a perfect matching of a connected cubic graph G such that the associated graph.G_m is equitable bipartite, with equitable bipartition (A,B).Let A_m(resp.B_m) be the vertices of G that are contained in the edges corresponding to the vertices of A(resp. B).

Consequently, A_m and B_m are two disjoint applied of critical exact doubly dominating sets of G.

IV.APPLICATION OF DOUBLE DOMINATION CRITICAL TREES AND CYCLES

Often when it is difficult to characterize graphs with particular parameters, it ishelpful to restrict one’s attention to trees.It has been found that no tree is dominationor total domination edge critical.

First we must define a star and double star. A star is a tree with exactly one vertex that is not a leaf.

Where $\gamma_2(K_{1,6})=7$ while $\gamma_2(K_{1,6}+e)=6$ for any edge $e \notin E(K_{1,6})$. In general,for the star $K_{1,m}$, where $\gamma_2(K_{1,m}) = m + 1$, then

$\gamma_2(K_{1,m} + e) = m+1$ for any edge

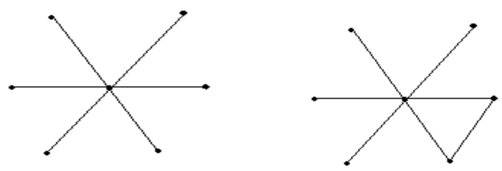
$e \notin E(K_{1,m})$. A double star is a tree with exactly two vertices that are not leaves in figure 2 .Using this information we characterize the double domination edge critical trees.

Proposition :1

A tree T is double domination edge critical if and only if T is a star or a double star.

Proposition :2

Let $G' = G + uv$ for any $uv \in E(G)$ and again the darkened vertices represent a double dominating set. Note that any edge added will decrease $\gamma_2(G)$ by one.



Let G be a star graph of order 7.

$\gamma_2(G)=7$

When an edge is added to the star graph, we have $\gamma_2(G+uv)=6$.

V.CONCLUSION

We discussed double domination, exact double domination and connected cubic double domination of graph and we established the necessary and sufficient condition of the existence of connected cubic double domination of graph with different parameters and also we verified that $\gamma_2(G)$ will decrease by one when any edge is added .

REFERENCES

[1] M.Blidia, M.Chellai, T.W. Hayes and M.Henning, Independent and double domination in trees, to appear in UtilitasMathematica.
[2] M. Blidia, M. Chellali and T.W. Haynes, Characterizations of trees with equal paired and double domination numbers, submitted for publication.
[3] M.Faber, Domination, independent domination and duality in strongly chordalgraphs, Discrete Appl. Math 7. (1984) 115-130.
[4] F.Harary and T.W.Haynes, Double domination in graphs, ArsCombis. 55 (2000) 201-213.
[5] T.W.Haynes, S.T. Hedetniemi and P.J.Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, New york, 1998).