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Mathematical Model of Cellular Automata and Fractals in Cancer Growth

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Abstract - In this paper, Mathematical Model can generate dynamics using Cellular Automata(CA) and Fractals in cancer growth. This model describes a cancer growth are inspired from the conway's game of life and each cell of the grid represents the location of living cells. To achieve this model we have developed an algorithm based mainly on Cellular Automata (CA) and Fractal Technique (FT). Using these both technique provide more information about cancer growth.

Keywords: Dynamics, Dimensions, Cellular Automata(CA), Fractals, Self-Similarity, Pascal Triangle and Sierpinski Triangle.

I. INTRODUCTION

Cancer is a disease which is made up of many types of cells. These cells are grow and are controlled to produce more cells. When cells become old (or) damaged they die and are replaced with new cells. Sometimes this process of controlled production of cells goeswrong. The genetic material(DNA) of a cell start producing mutationsthat affect normal cell growth and division by being damaged. When this happens, these cells do not die but form a mass of tissue called a tumor.

II. CELLULAR AUTOMATA(CA)

Usually, common cell shapes are squares, cubes and hexagons. But the simplest Cellular Automata(CA) are one-dimensional, with cells on a straight line and each cell can have only two possible states (high/low (or) black/white). In theory, a Cellular Automata can have any number of dimensions and each cell can have any number of possible states. The state of each cell changes in discrete steps at regular time intervals.
Cellular Automata(CA) in Computer Modeling:
The special computer program on the monitor screen as small squares, triangles or other shapes which are called "cells". Each cell is connected to its neighboring cells by a set of simple rules and the program begins usually with one or a few cells which trace a simple and often predictable pattern.

III.PRELIMINARIES ON TOPOLOGICAL DYNAMICS OF CELLULAR AUTOMATA (CA)

General Dynamical System: A dynamical system is a couple (X,F), where X is a Topological space and F is a Self-map on X.

A Cellular Automata (CA): It is a dynamical system (A^z, F) such that Foσ=σoF, that is the transition rule of a Cellular Automata(CA) commutes with shift map.

Shift Map: Dynamical systems are discretized with respect to time and we study then as discrete dynamical systems basically defined by maps known as shift map.

Attracting and Repelling Fixed Points: Cellular Automata (CA) evolve after a finite number of time steps from almost all initial states to a unique homogeneous state, in which all sites have the same value such Cellular Automata may be considered to evolve to simple "Limit points" in phase space. These limit points and which all sites are attracted towards are called attracting fixed points. If the sites repel away from fixed point, then those points are known as "repelling fixed points".

Example:1 f(a b c) = (b+c) (modulo 2), A= { 0,1}
Local Rule Table:

A	b	c	F(a,b,c)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0

Example:2f(a b c) = Max {A, b}, A= {0,1,2}
Local Rule Table:

A	b	c	F(a, b,c)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1

For different initial configuration, different Cellular Automata(CA) can be generated.

Simple Cellular Automata in Pascal Triangle:
Cellular Automata have an even larger history. In some sense we might say that Pascal Triangle is the first Cellular Automata. It sets a good example for Cellular Automata. Pascal Triangle made up of staggered rows of numbers.
The initial condition is used are,
$a_{i1}= 1$
$a_{ii}= 1$
where, i: the row number.
The cells in the Pascal Triangle are generated using the law.
$a_{ij} = a_{(i-1)(j-1)}+a_{(i-)j}$
where, i: the row number
j: the column number.

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1					
1	1				
1	2	1			
1	3	8	1		
1	4	6	4	1	
1	5	10	10	6	1

Fig:1Pascal Triangle

On reducing this Pascal Triangle to modulo 2.
It takes the form,

1					
1	1				
1	0	1			
1	1	0	1		
1	0	0	0	1	
1	1	0	0	0	1

Fig:2Pascal Triangle modulo 2

On replacing one's by dot and zero's by blank space.

0					
0	0				
0		0			
0	0	0	0		
0				0	
0	0			0	0

Fig:3Pascal Triangle-A modification

A beautiful aspect of Pascal Triangle modulo 2 is that the pattern n inside any triangle of points is similar in design to that of any sub-triangle, though larger insize.
If we extend Pascal triangle to infinitely many row and reduce the scale of our picture in half each time that we double the number of rows. Then the resulting design is self-similar known as "Fractals".

A Cellular Automata is a Fractal type of dynamical system.
Fractal Dimension – Introduction:

Every one knows the dimension of a line, a square and a cube. They are one, two and three respectively. And, we can measure the distance, area and volume of those objects as well. However, what is the dimension of the inside of a kidney or the brain and how do we measure their surface area? How about a piece of cauliflower? This is where Fractal dimension can help us out. Fractal dimension allows us to measure the complexity of an objects.

There are two types of Fractal Dimensions. They are,

1. Self-Similarity Dimension
2. Box-Counting Dimension

Self-Similarity Dimension:
A set is called Self-Similar. If it can be broken into arbitrary small pieces, each of which is a small replica of the entire set.

Example:

1. We may break a line segment into four Self-Similar intervals. Each with the same length and each of which can be magnified by a factor of four to yield the original segment.
2. We can decompose a square into four Self-Similar sub-squares and the magnification factor is two.
3. Also, We can decompose a cube to N^3 Self-Similar pieces, each of which has magnification factor is N.

Fractal Dimension Formula:
Fractal Dimension Formula is given by,

$$\text{Fractal Dimension} = \frac{\log(\text{number of Self-Similar pieces})}{\log(\text{magnification factor}}$$

For Square,

$$\text{Dimension} = \frac{\log N^2}{\log N} = \frac{2\log N}{\log N} = 2$$

Similarly, the dimension of a cube is as follows:

$$\text{Dimension} = \frac{\log N^3}{\log N} = \frac{3\log N}{\log N} = 3$$

This dimension is different from the Topological dimension. (i.e.) the mathematical dimension.

SierpinskiTriangle(ST):
The mid-points of each side of an equilateral triangle and connecting them together, omitting the middle-most triangle one gets on interesting Fractal known as the "Sierpinski Triangle". The very simple Fractal arise

Example:
We compute the Fractal dimension of "Sierpinski Triangle". The Sierpinki Triangle can be decomposed into 3 Self-Similar pieces each with magnification factor 2.

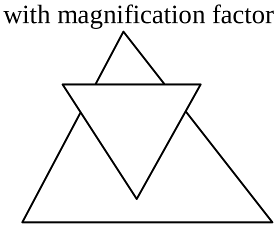

Fig:4Sierpinski Triangle with Magnification factor 2.

$$\text{Fractal Dimension} = \frac{\log(\text{number of Self-Similar pieces})}{\log(\text{Magnification factor})} = \frac{\log 3}{\log 2}$$

Volume: 03 Issue: 03 December 2014 Pages: 243-245
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$$\approx 1.58$$

So, the dimension of S is somewhere between 1 and 2.

Also, S can be decomposed into 9 Self-Similar pieces with magnification factor 4.

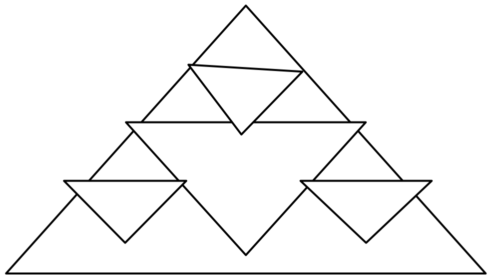

Fig:5Sierpinski Triangle with Magnification factor 4.

Now, We have

$$\text{Fractal dimension} = \frac{\log(\text{number of Self-Similar pieces})}{\log(\text{Magnification factor})}$$

$$= \frac{\log 9}{\log 4}$$

$$= \frac{\log 3}{\log 2}$$

$$\approx 1.58$$

Similarly, S can be decomposed into 3^N Self-Similar pieces with magnification factor 2^N.

So,

$$\text{Fractal dimension} = \frac{\log 3^N}{\log 2^N}$$

$$= \frac{N \log 3}{N \log 2}$$

$$= \frac{\log 3}{\log 2}$$

$$\approx 1.58$$

The larger Fractal dimension, the more object fills up the space. For example "Koch Curve". It fills up more space than a simple line, but it does not cover a complete 2-D area. Hence Fractal dimension can be seen as an index of the space filling property of a Fractal object.

Evolving Cellular Automata (CA) to simulate cancer growth:

Consider,any cancer patient, cancer cells and other cell positions can be monitored. Two-Dimensional Cellular Automata models were designed and implemented in order to simulate cancer growth. A two-dimensional M-rows, N-columns (M×N) grid is implemented, each cell of the grid represents the location of a living cells.

The cells obtains one of the following values.

N – Normal cell
C – Cancer cell in the reproduction phase
c – Cancer cell
D – Dead cell

The dynamics of the Cellular Automata can be described by the following rules:

1. N → c
   A normal cell turns to a Cancer Cell.
2. c →C
   Cancer Cell turns to the reproduction cell.
3. C→ 2c
   Reproduction cell becomes 2 times cancer cell
4. c→D
   A cancer cell dies
5. N→D
   A normal cell dies
6. D →N
   A dead cell location is occupied by a normal cell.
7. D →C
   A dead cell location is occupied by a cancer cell.

Application in Fractal Dimension:

Fractal Technique have been applied to cancer growth of tumors might be influenced by the Fractal structure of their tissues of origin. Tumor boundaries and chromatin texture have been studied by analysis. This may prove useful in discriminating between "benign and malignant cells".

IV.CONCLUSION

The world of Mathematicians and Physicists have overlooked dynamical system as random and unpredictable. There are various tools used to gain the comprehensive overview of the dynamics Cellular Automata and Fractal. The dynamical system Cellular Automata and Fractals is used as a computational tool. Using the tools, we evolve the simulation of cancer growth which is compare with theoretical, Mathematical models.

REFERENCES

[1] TommasoToffoli, 1984 "Cellular Automata as an Alternative to Differential Equations".
[2] S. Wolfram, 1986, "Theory and Application of Cellular Automa".
[3] P.Kurka, 2001, "Topological Dynamics of Cellular Automata, Codes systems, and graphical models".
[4] B.BondarehKo, 1990,"Generalized Pascal Triangle and pyramids, their Fractals, Graphics and Applications".
[5] Ary L. Goldberger, 1997, "Fractal Variability VersusPathalogic Periodicity".
[6] P.Anand and A.B. Kunnumakara, Cancer is a preventable disease that requires major lifestyle changes, Pharm. Res.25(9)(2008),2097-2116.
[7] M.Mamat, E.S Nugraha, and A,Kartono. Mathematical modeling of tumor immune interaction. Far East Journal of Applied Mathematics. 41(1)(2010), 133-151.